Fractionalization in quantum matter: past, present and future

1) Unraveling the hidden link between composite fermions and exciton condensate

2) Quantum oscillations in insulators with neutral Fermi surfaces

IMPRS RETREAT

September 20, 2017

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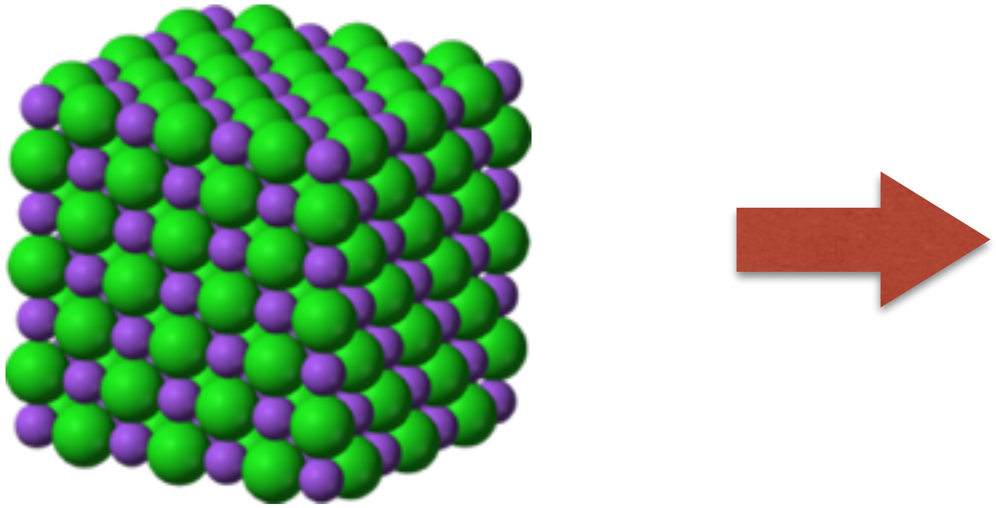
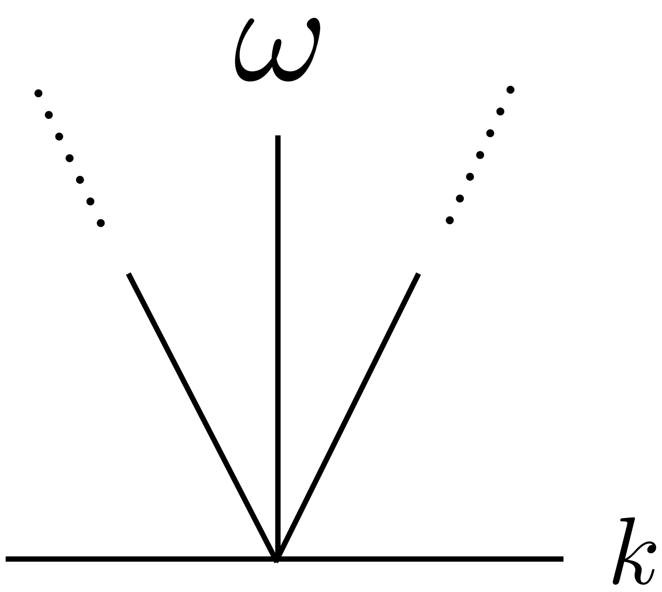
Max Planck Institute for the Physics of Complex Systems

Debye and the birth of quasiparticles

Debye

- Debye model (1912)

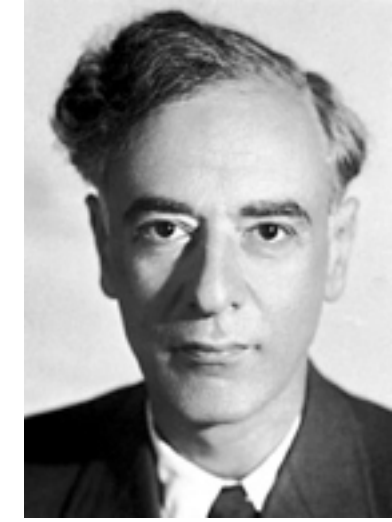




- Quantized a la Planck-Einstein black body photons.
- Sound ~ Light. Phonon ~ Photon.

Bohr model (1913)   Bose paper (1924)

Landau and the quasiparticle paradigm



Landau

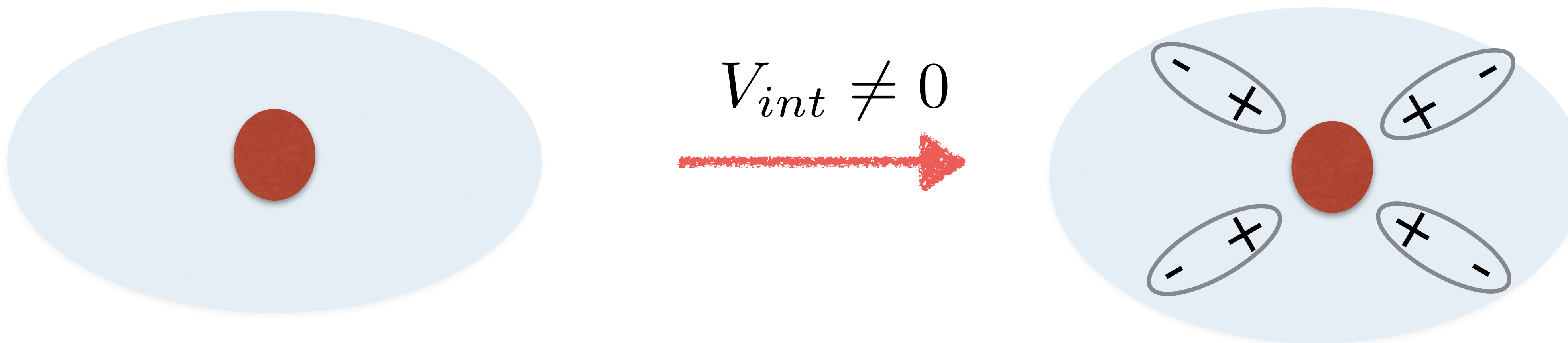
- Charged quasiparticles: Fermions.

$\Delta Q = +1$ $\Delta Q = +1$

$t = -\infty$

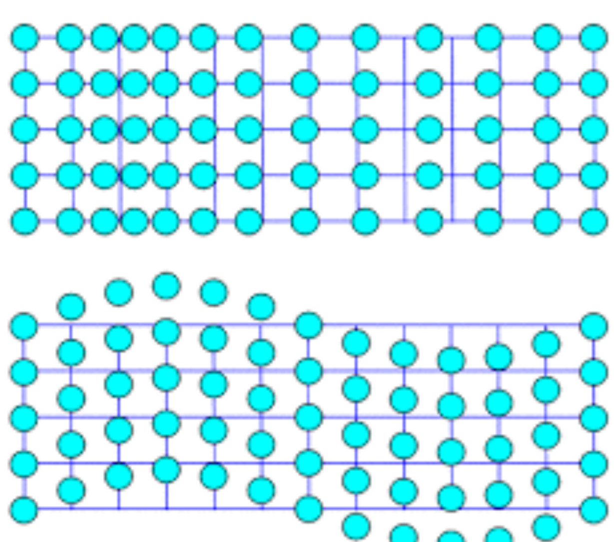
$V_{int} = 0$

$V_{int} \neq 0$

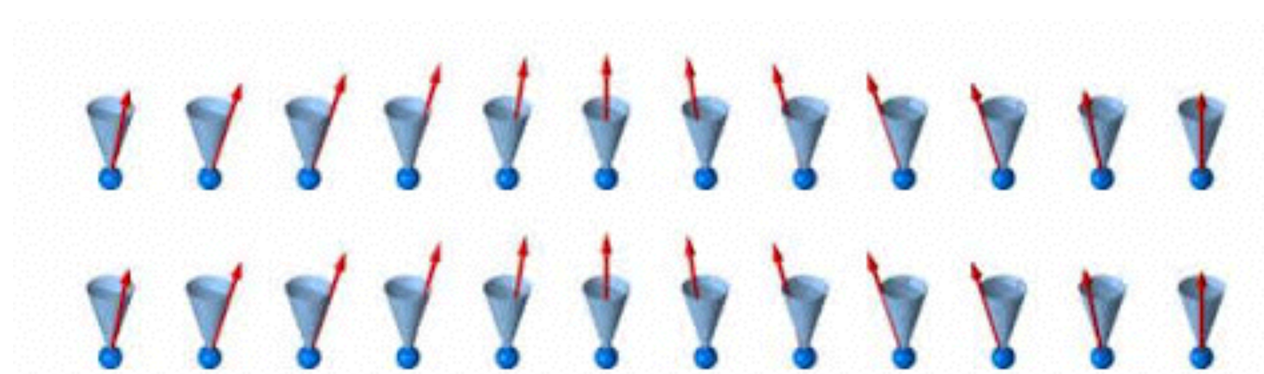


- Neutral quasiparticles (quanta of collective oscillations): Bosons.

Phonon
Sound

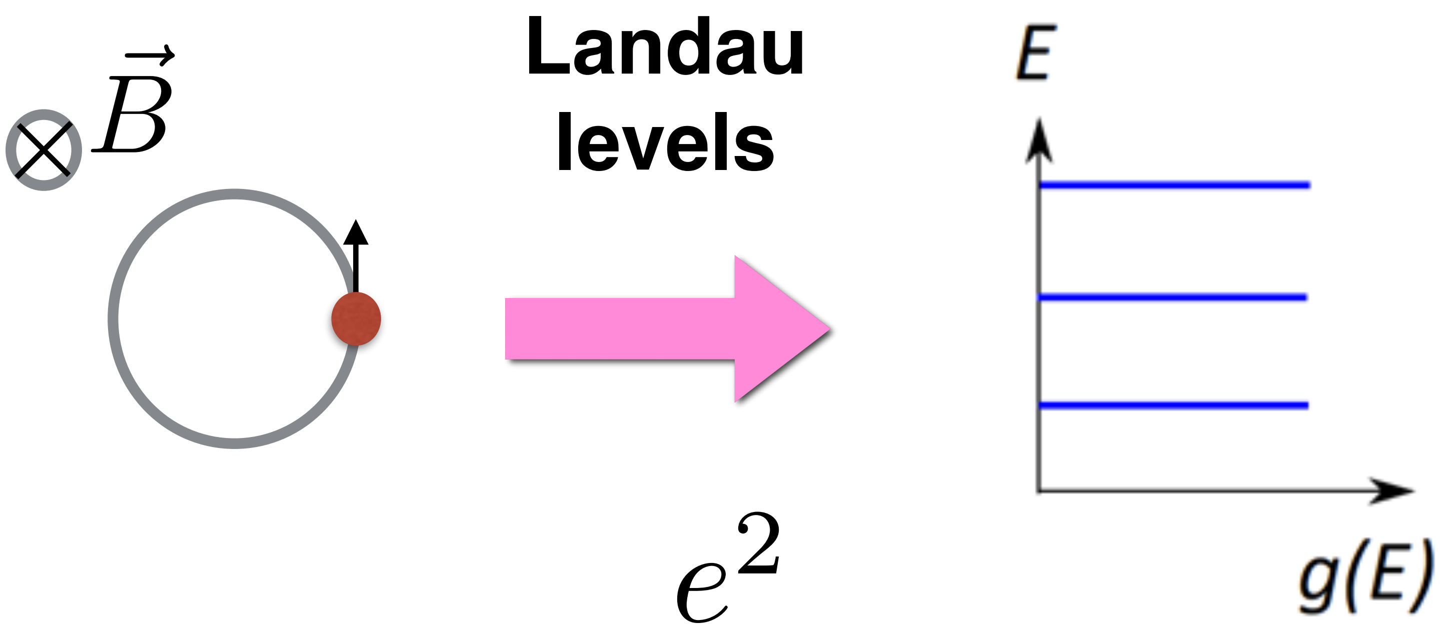


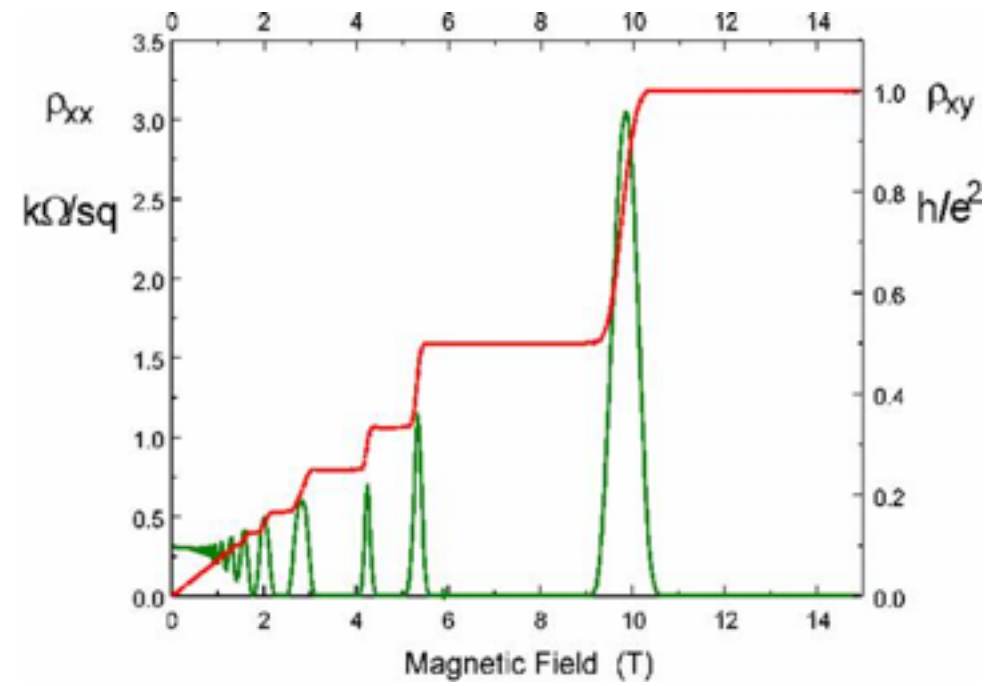
Magnon Spin waves



Quantum Hall revolution

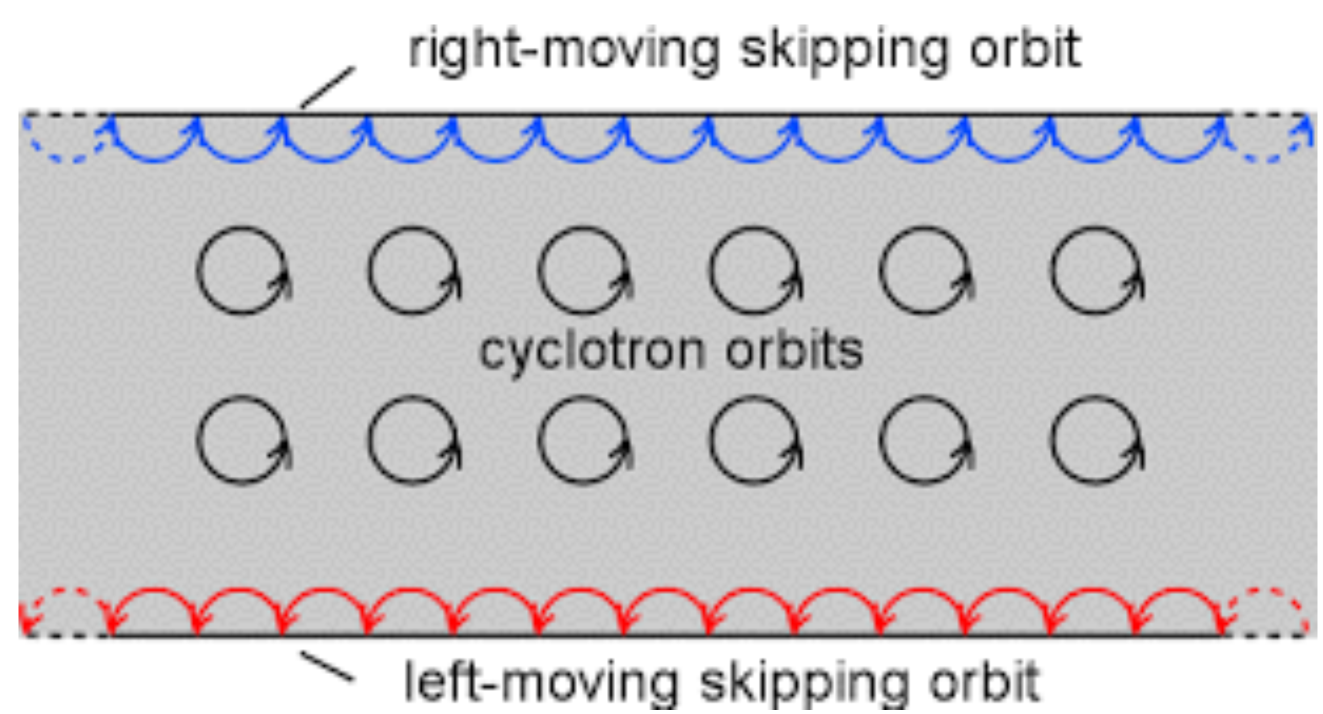
- Electrons under strong magnetic fields display the quantum Hall effect:





$$\sigma_{xy} = n\frac{e^2}{h}$$

$$n = 1, 2, 3...$$



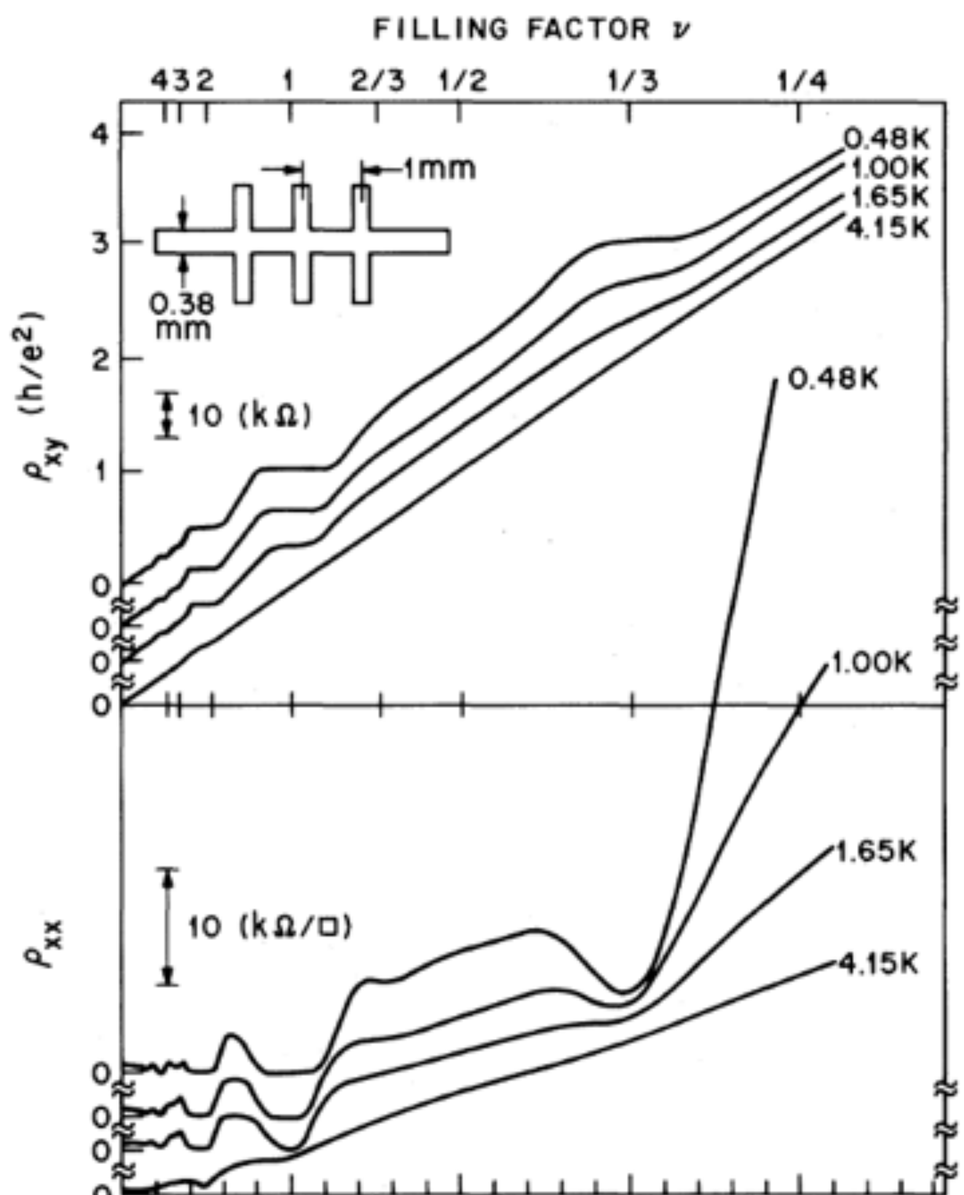
Landau level filling:

$$\nu = \frac{N_e}{N_\phi} = n \qquad N_\phi = \frac{BA}{\Phi_0}$$

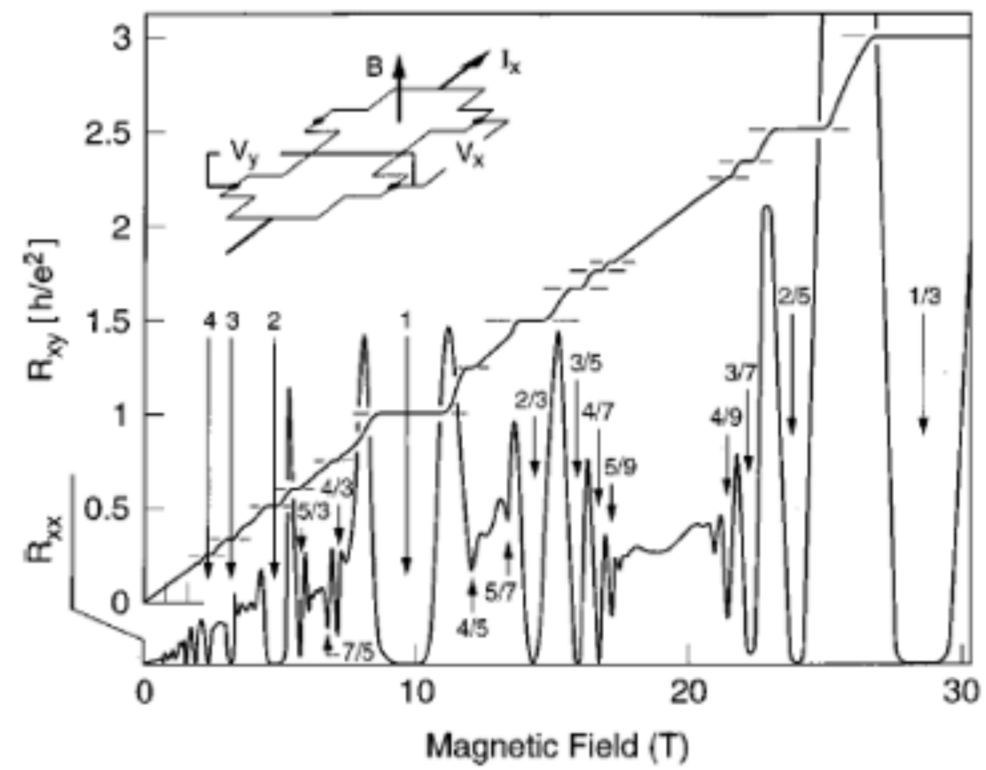
Fractional Quantum Hall effect

Stormer Tsui Gossard

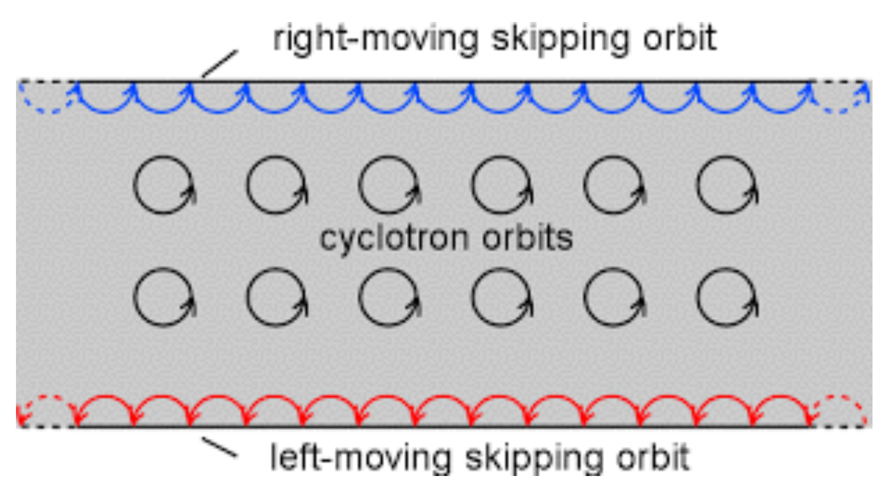
- Plateau at 1/3?



Tsui, Stormer, Gossard, PRL (1982).



Eisenstein, Stormer, Science (1990).

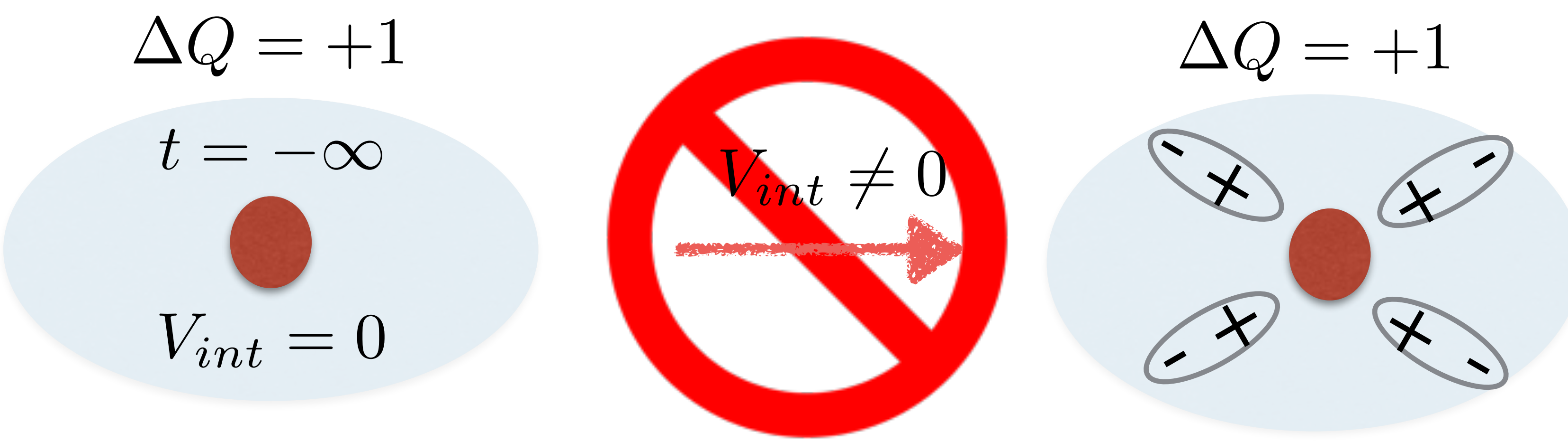


$$\sigma_{xy} = \frac{e^2}{3h}$$

???

Fractional Quantum Hall effect

Laughlin

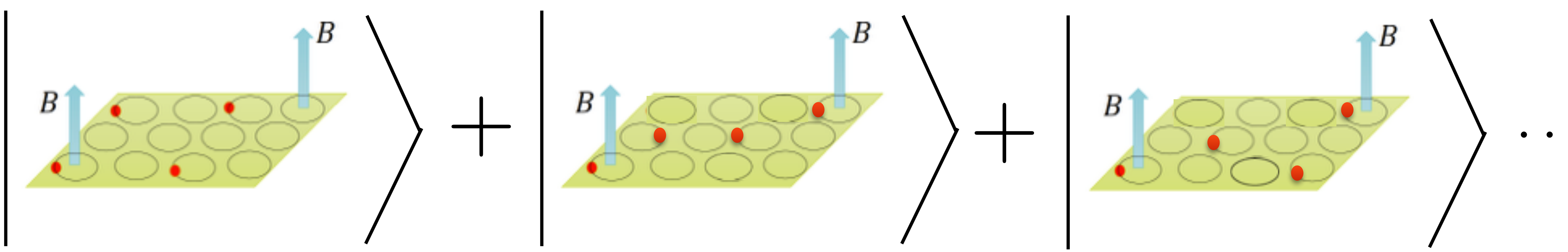


- Laughlin liquid at filling 1/3

$$\Psi = \prod_{i<j} (z_i - z_j)^3 e^{-\frac{|z_i|^2}{4l^2}}$$

$$z = x + iy$$

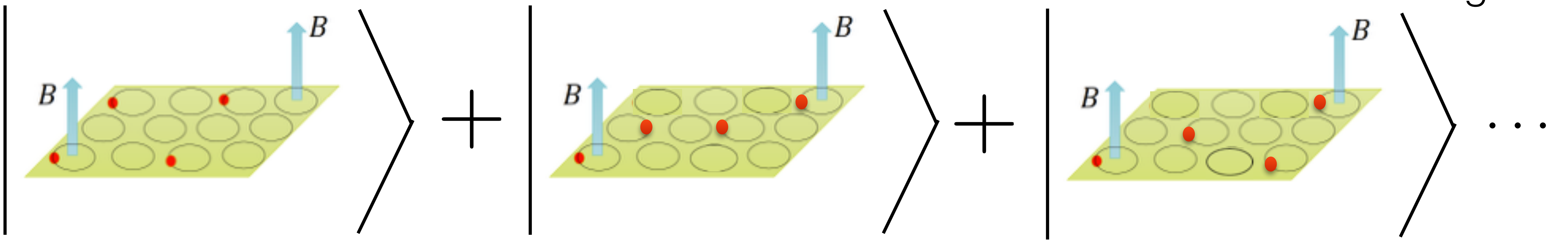
$$\nu = \frac{N_e}{N_\phi} = \frac{1}{3}$$



Fractional Quantum Hall effect

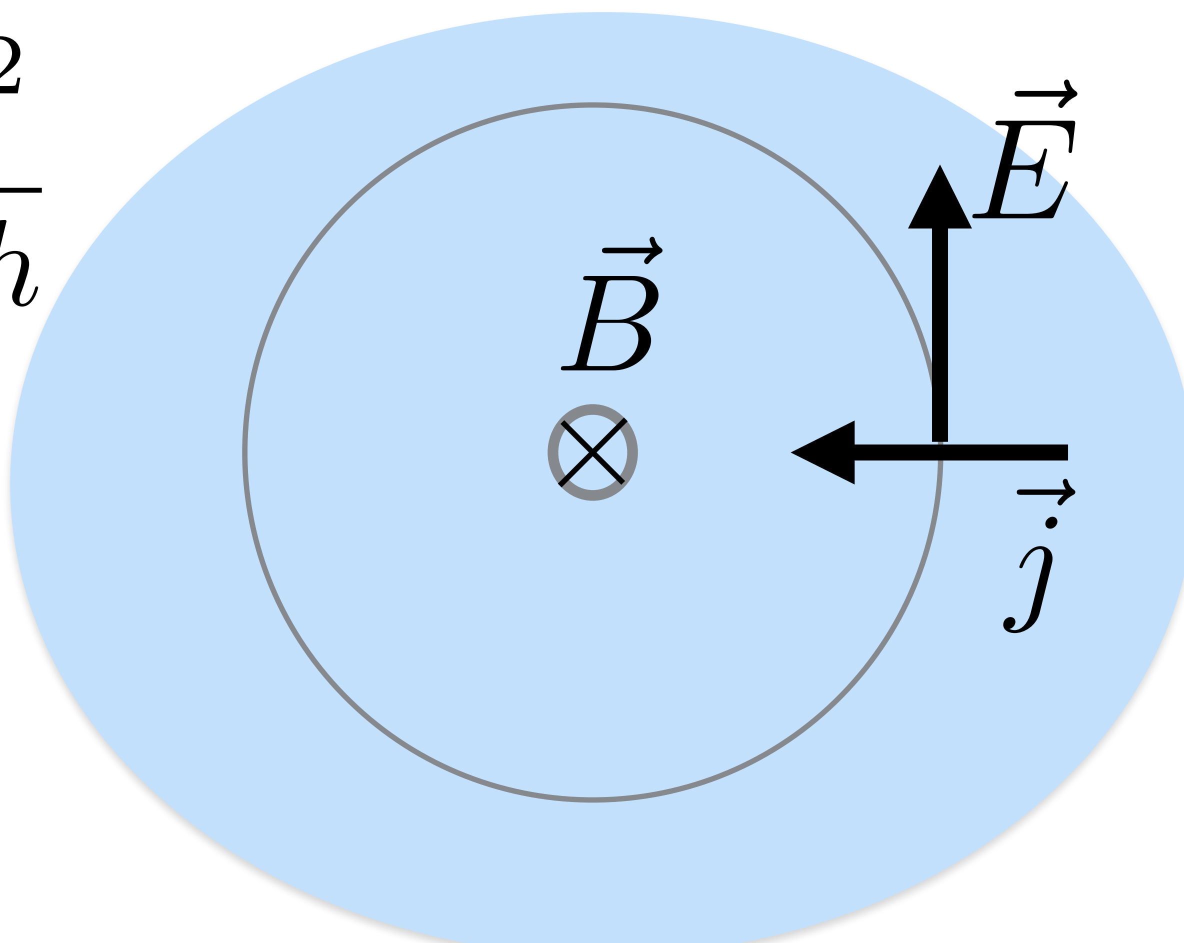
Laughlin

- Laughlin liquid at filling 1/3 $\nu = \frac{N_e}{N_\phi} = \frac{1}{3}$



$$\sigma_{xy} = \frac{e^2}{3h}$$

$$\Phi_0 = \frac{h}{e}$$



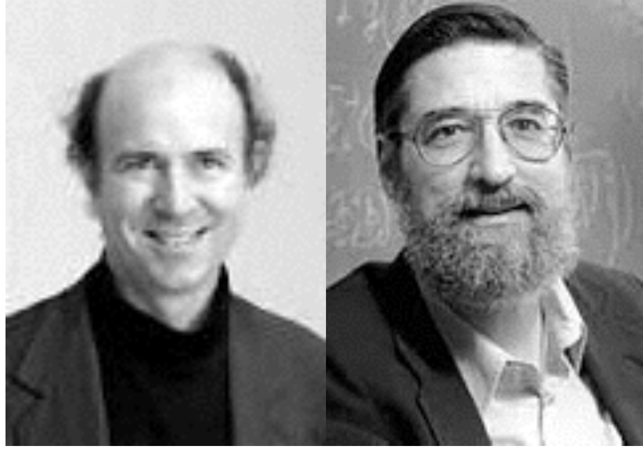
$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$j_x = \sigma_{xy} E_y$$

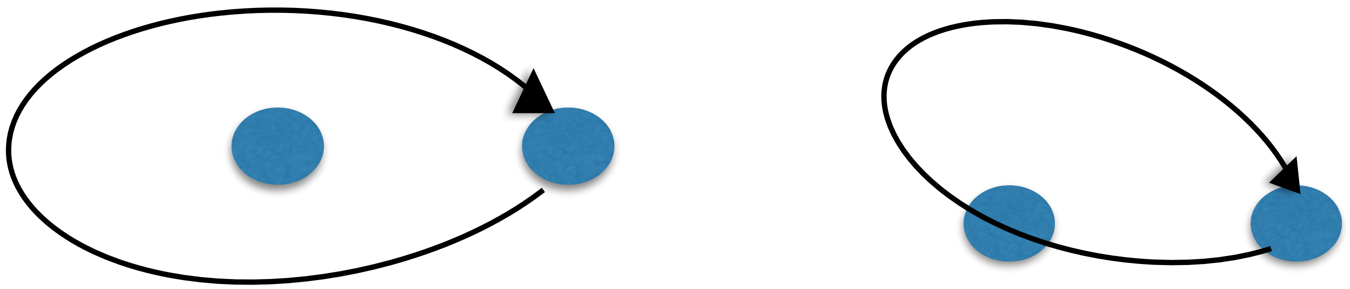
$$\Delta Q = \sigma_{xy} \Delta \Phi$$

$$\Delta Q_0 = \pm \frac{e}{3}$$

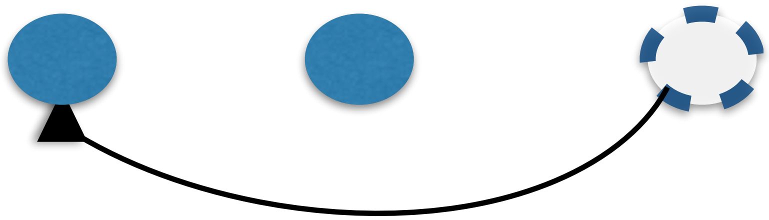
Fractional Statistics


Wilczek Halperin

- Only fermions and bosons in 3D:



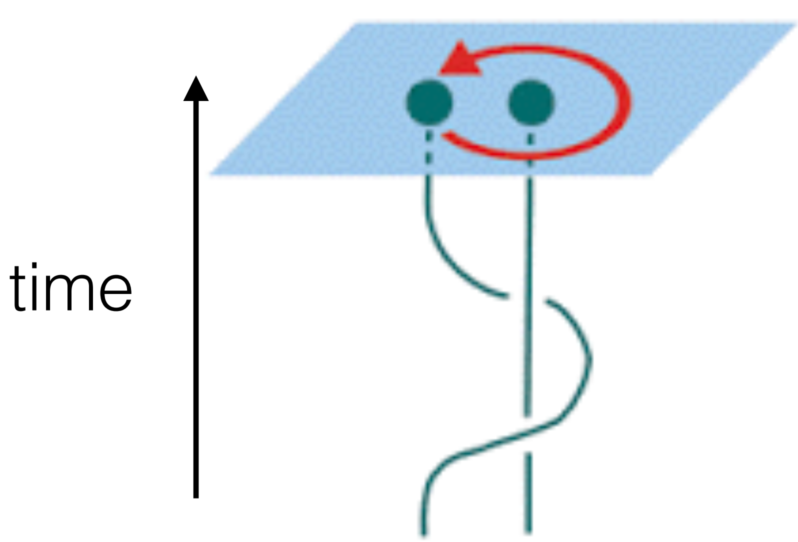
$$(e^{i\varphi})^2 = 1$$



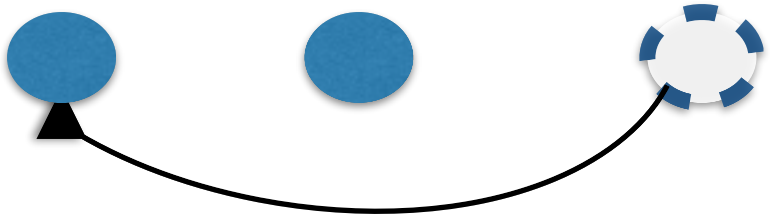
$$e^{i\varphi} = \pm 1$$

Bosons or Fermions

- In 2D "any-ons" are allowed:



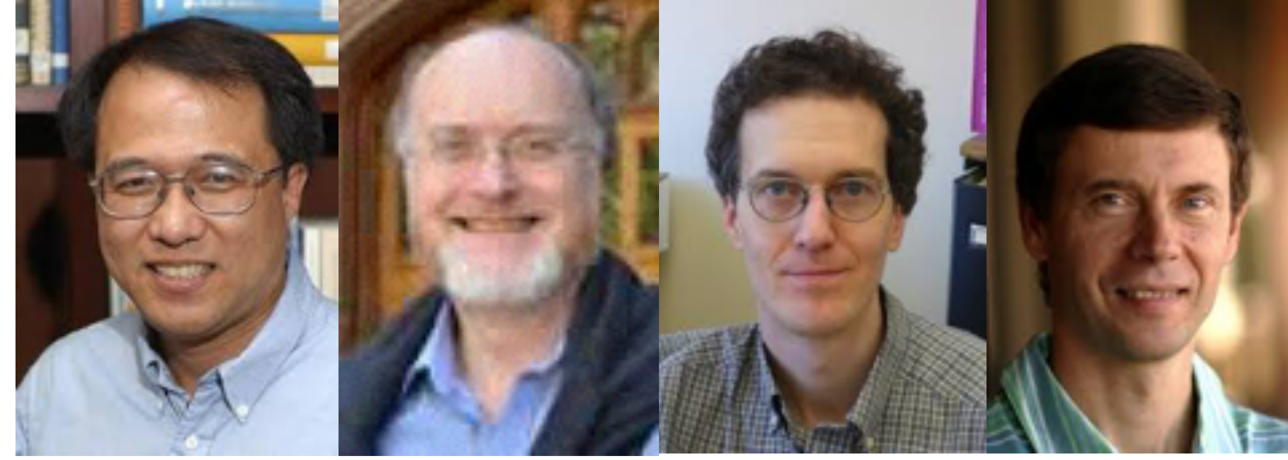
$e/3$ $e/3$



$$\varphi = \pi/3$$

Laughlin anyons

Fractionalization and topology

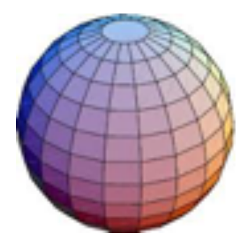


Wen Read Moore Kitaev

- Non-trivial degeneracy on closed manifolds:

$$\mathcal{D} = (\text{nontrivial quasiparticles} + 1)^G$$

$G = 3$ 



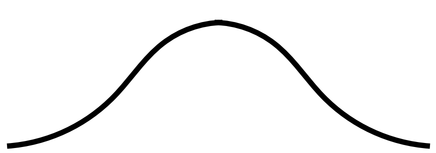
$G = 0$

1

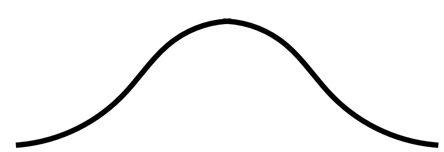
$G = 1$

non-trivial qps+1

- Non-abelian anyons: “irrational” size of Hilbert space.



γ_1



γ_2

$$D_{\gamma_1 \gamma_2} = 2 \rightarrow D_\gamma = \sqrt{2}$$

Majorana fermions

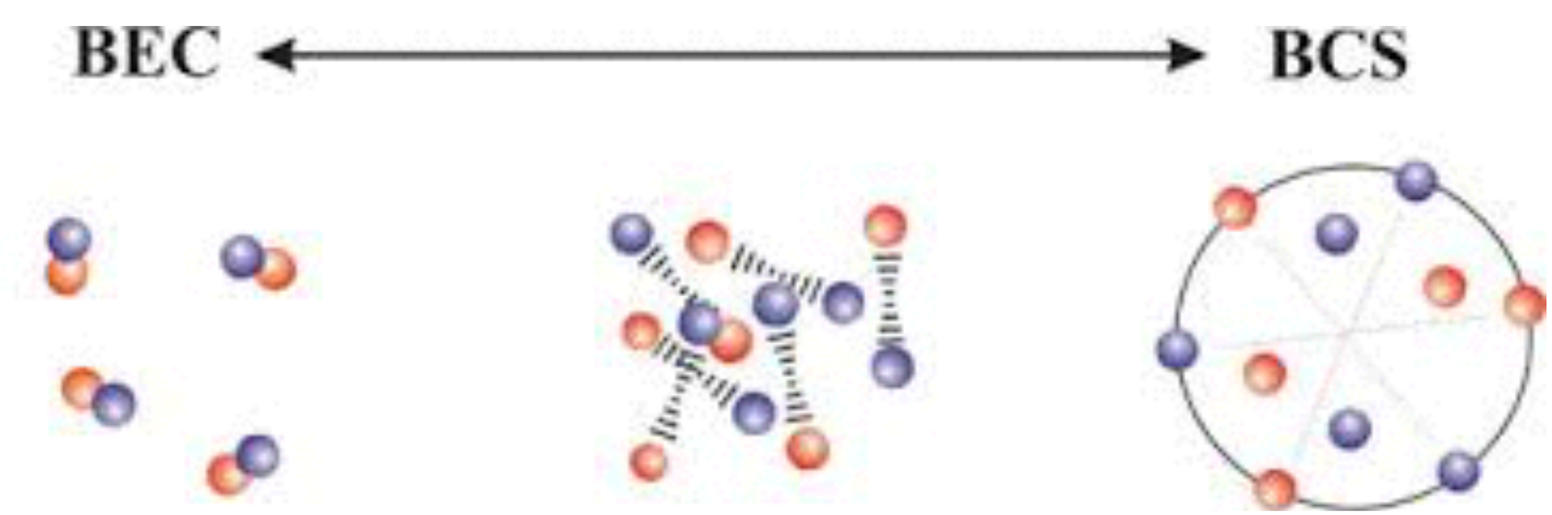
Experimentally realized in:

-GaAs at $\nu = 5/2$

-1D chains superconducting p-wave.

The hidden link between composite fermions and the exciton condensate

- BEC - BCS crossover a powerful unification in physics of quantum matter:



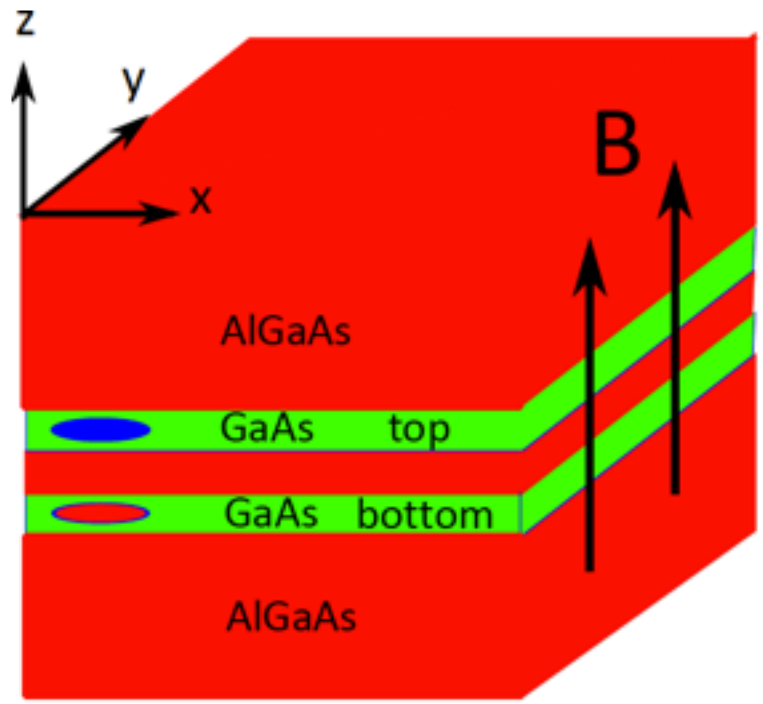
Bose Einstein condensate molecules

Superconductor Fermions

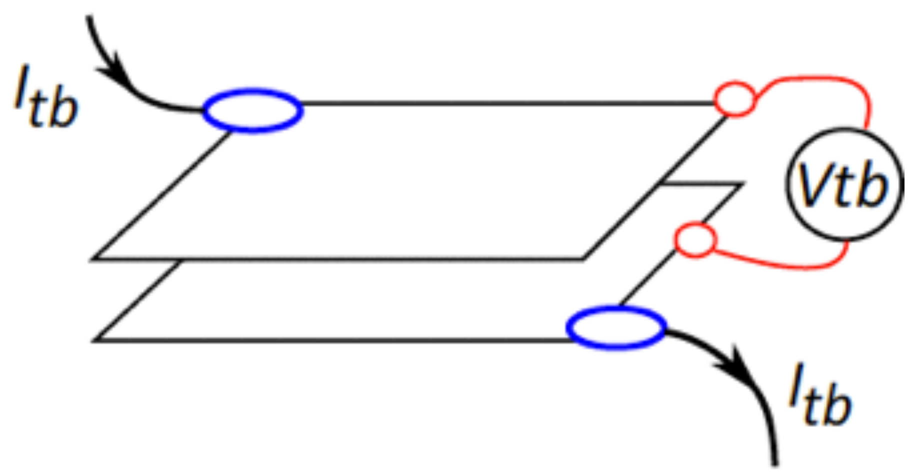
- Unification between two celebrated quantum Hall phases of matter: the **exciton condensate** and the **composite fermion metal.**

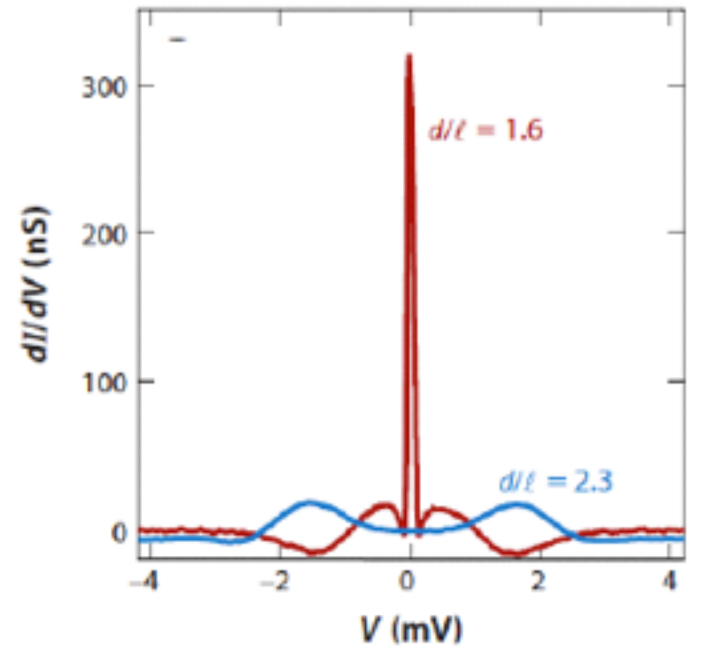
Exciton condensate

- No tunneling but strong interactions

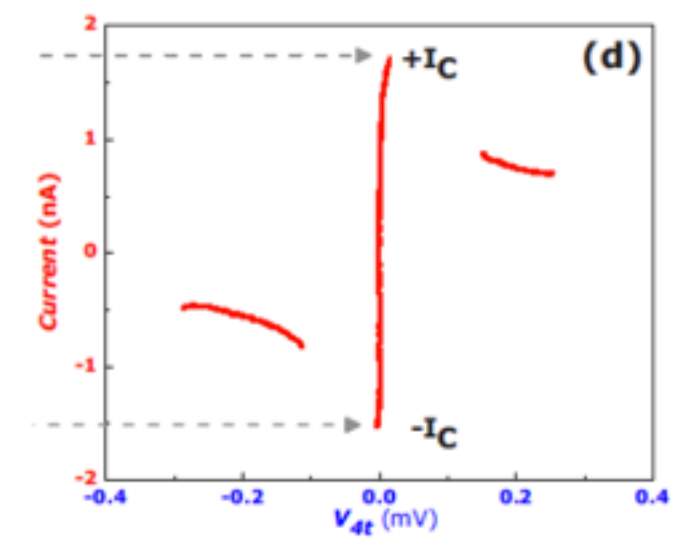


$$\nu = \nu_{top} + \nu_{bottom} = 1/2 + 1/2$$



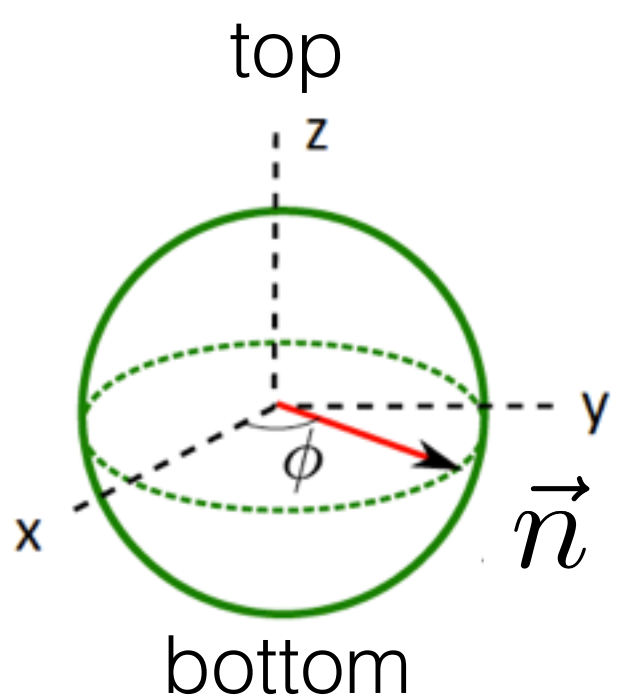


Spielman *et al.*, PRL (2000)



Tiemann *et al.*, PRB (2007)

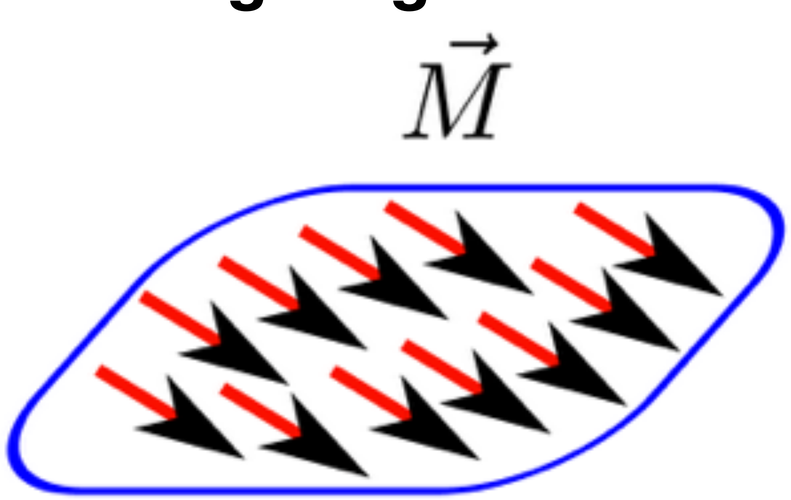
- Exciton condensate:



$$|top\rangle + e^{i\phi}|bottom\rangle$$

$$\langle c^{\dagger}_{\text{bottom}} c_{\text{top}} \rangle \propto e^{i\phi}$$

Long range XY order

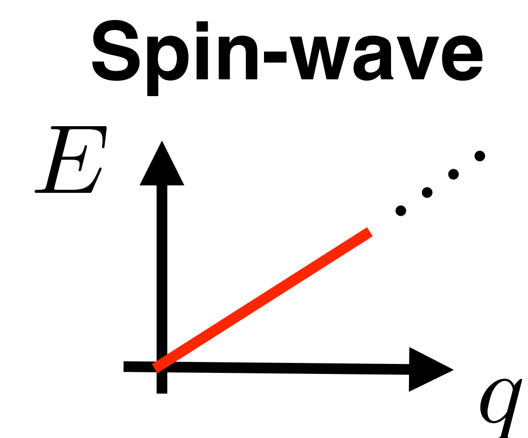


Properties of exciton condensate

- Superfluidity for charge imbalance:

$$Q_- = Q_{top} - Q_{bottom} \qquad [Q_-, \phi] = i$$

- Linearly dispersing Goldstone mode of ϕ (pseudo-spin wave).

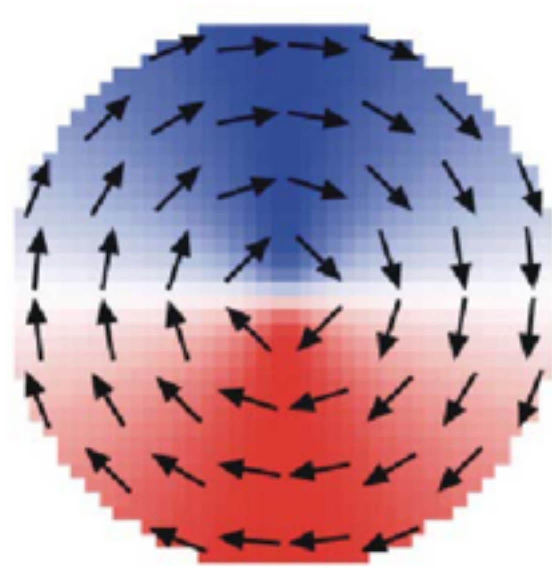


- Half-charged vortices (merons):

$v = 1$

$Q_+ = e/2$

2π winding



$$Q_+ = (v n_z)\,\frac{e}{2}$$

$v \in \mathbb{Z}$

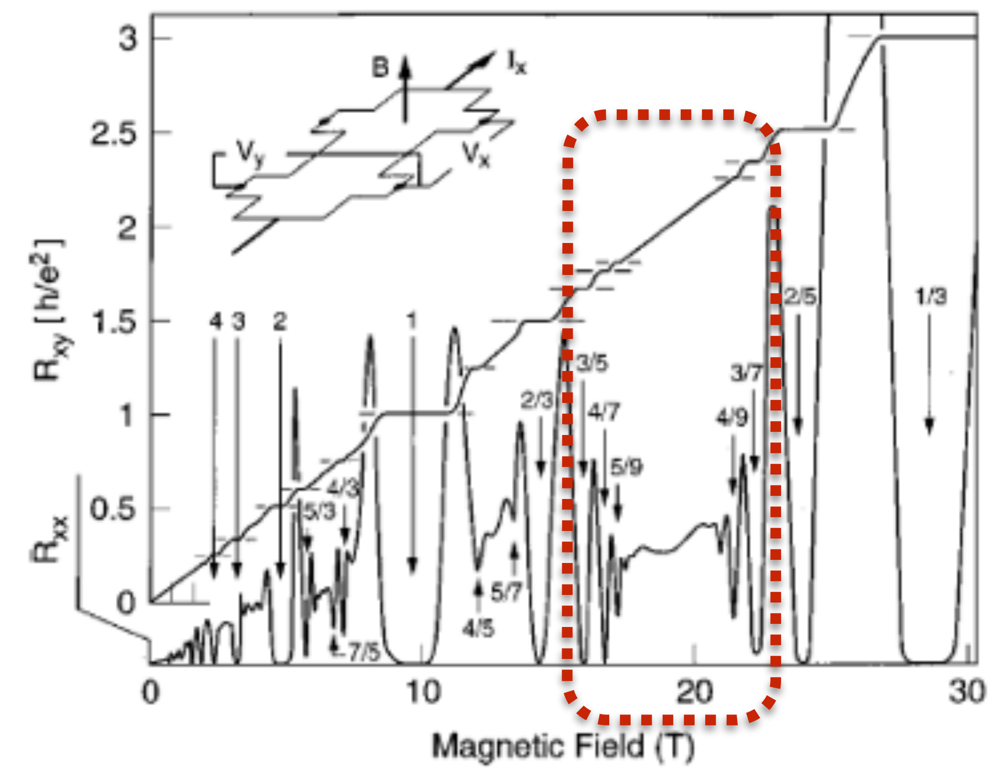
$n_z = \pm 1$

- Wen, Zee, PRL 69, 1811 (1992).
- Moon, Mori, Yang, Girvin, MacDonald, Zheng, Yoshioka, Zhang, PRB 51, 5138 (1995).

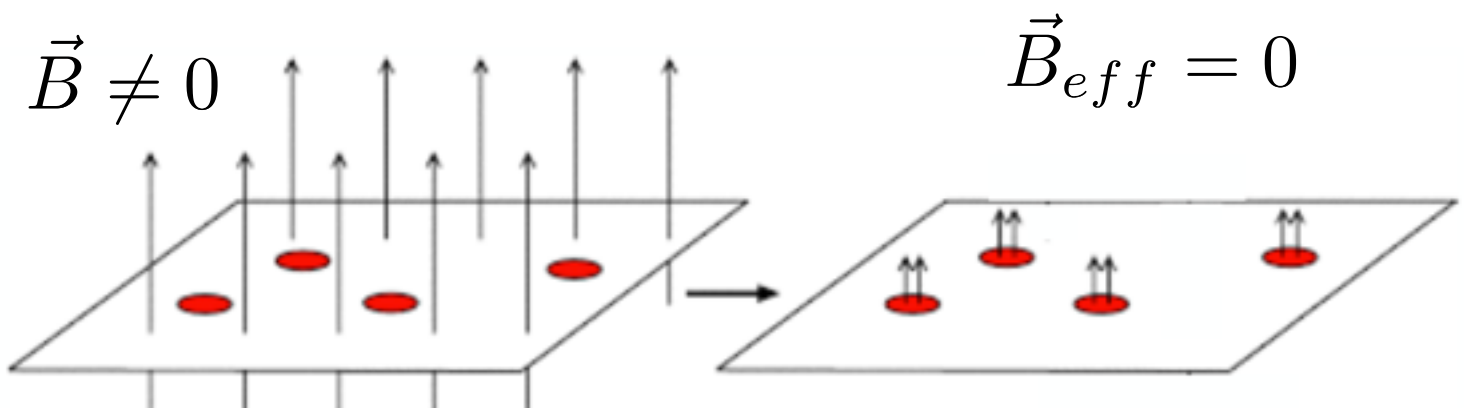
Composite fermion metal

- Fractionalized metal for half filled landau level:

$$N_e = \frac{1}{2} N_\phi$$

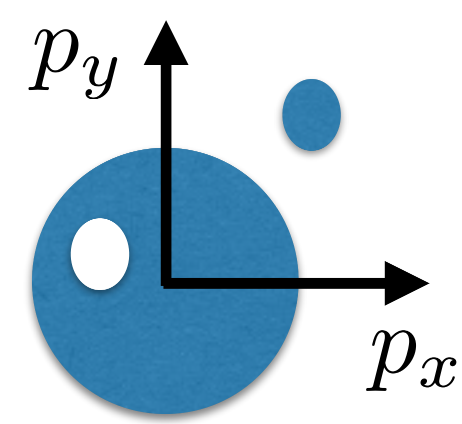


- Composite fermion: electron bound to two vortices



electrons

composite fermions



composite fermion fermi surface

- Emergent 2-dimensional "gauge field" (analogous to the electro-magnetic field in 2D).

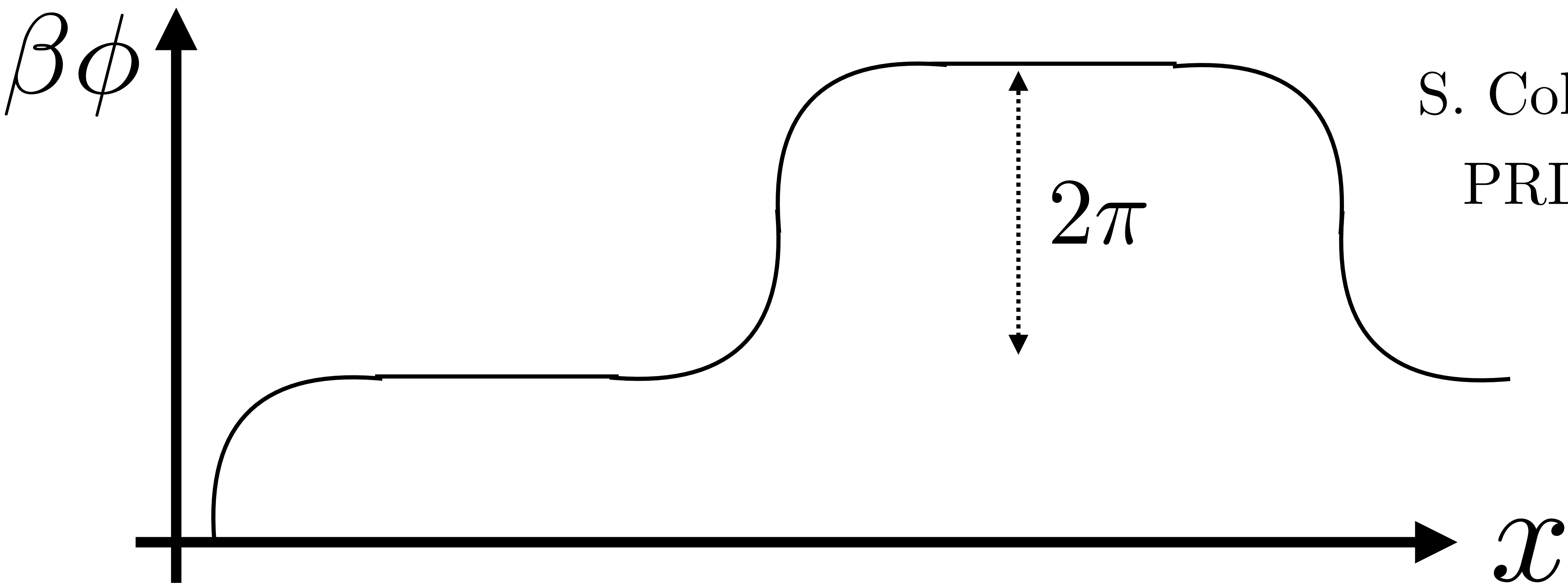
Jain, PRL (1989). Halperin, Lee, Read, PRB (1993).

Duality in 1+1D QFT's

$$1+1\ \mathrm{Sine-Gordon} \equiv 1+1\ \mathrm{Massive-Thirring}$$

$$\frac{1}{2}(\partial\phi)^2 + (m/\beta)^2\cos(\beta\phi) \equiv \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{g}{2}(\bar{\psi}\gamma_\mu\psi)^2$$

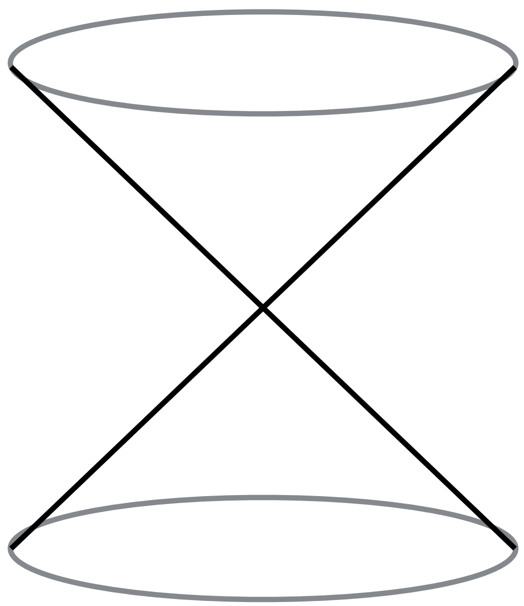
$$\frac{4\pi}{\beta^2} = 1 + \frac{g}{\pi} \qquad \psi^\dagger\psi = \frac{\beta}{2\pi}\partial_x\phi$$



S. Coleman, PRD 1975

Fermion vortex duality

Physical Dirac fermion

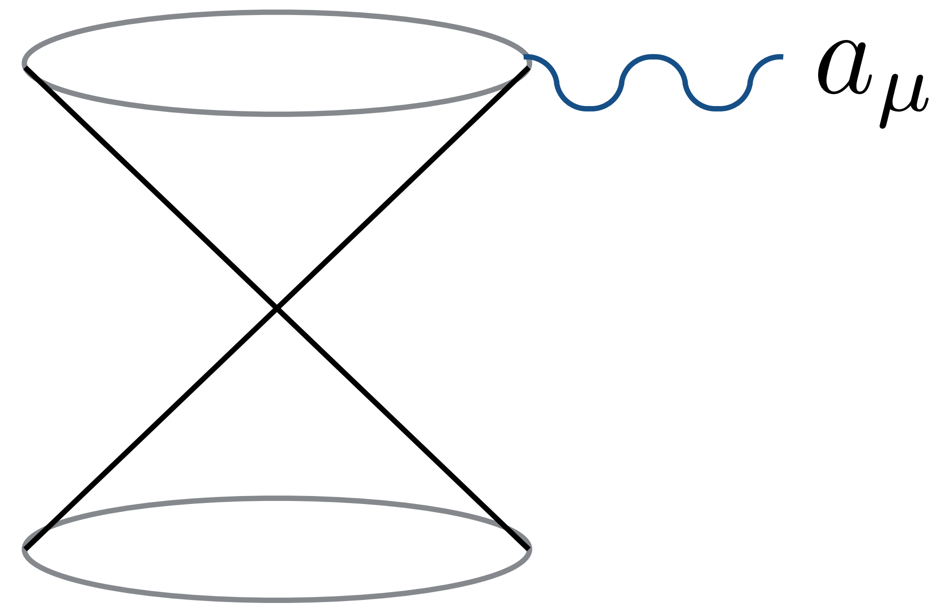


$$\mathcal{L}_e = \bar{\psi}_e(i\not{\partial} - \not{A})\psi_e + \mathcal{L}_{\text{int}}$$

$$\delta n_{elec}(r) = \frac{\nabla \times \vec{a}}{4\pi}$$

$$\psi_e^\dagger \leftrightarrow M_{4\pi}$$

Dirac composite fermion vortex



$$\mathcal{L}_{cf} = \bar{\psi}_{cf}(i\not{\partial} - \not{a})\psi_{cf} + \frac{adA}{4\pi} + \mathcal{L}_{\text{int}}$$

$$\hat{z} \times \vec{j}_{elec}(r) = \frac{\nabla a_0 + \partial_t \vec{a}}{4\pi}$$

Electron creation is flux insertion operator

Son PRX (2015). Wang, Senthil PRB (2016). Metlitski, Vishwanath, PRB (2015). Mross, Alicea, Motrunich PRL (2016). Seiberg, Wang, Senthil, Witten Ann. Phys. (2016).

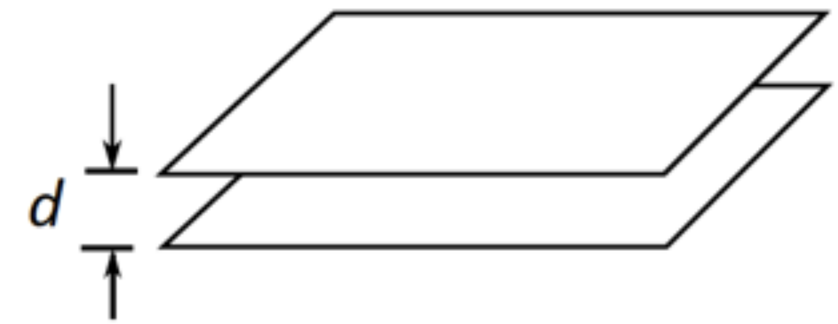
Bilayer exciton condensate and Composite fermion metal

- Are zero and infinite distance connected?

$$\nu = \nu_{top} + \nu_{bottom} = 1/2 + 1/2$$

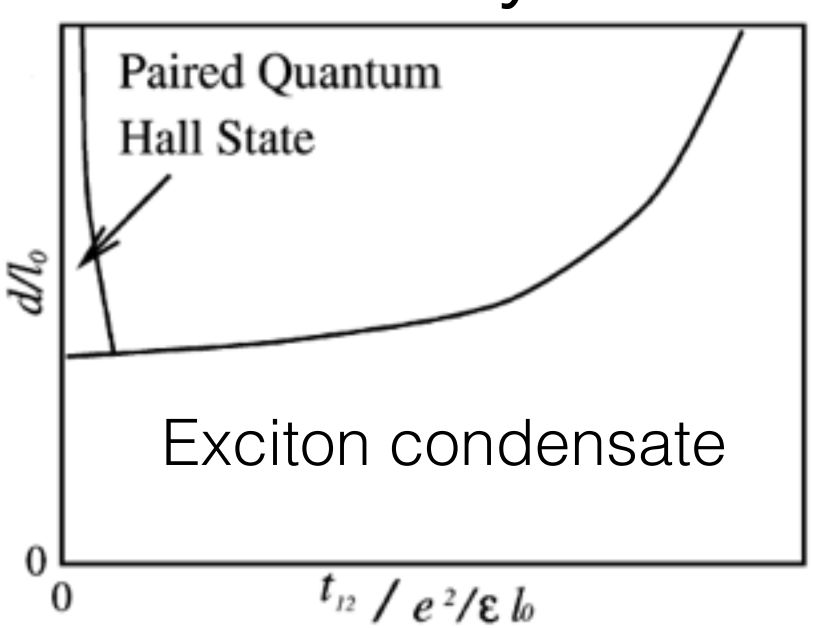
$d = 0$  $d = \infty$

Exciton condensate ? Two C-Fermion metals



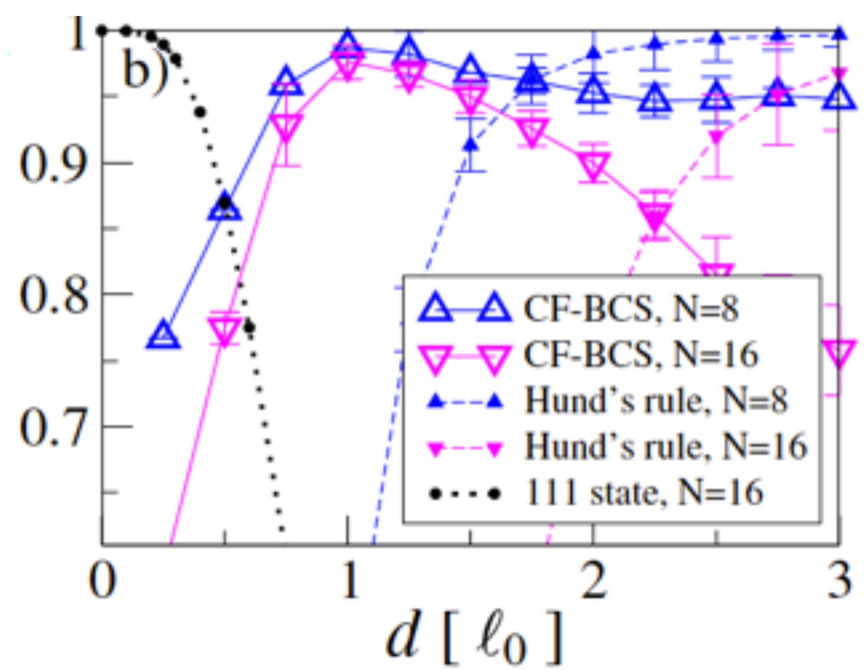
- Precedents

Theory



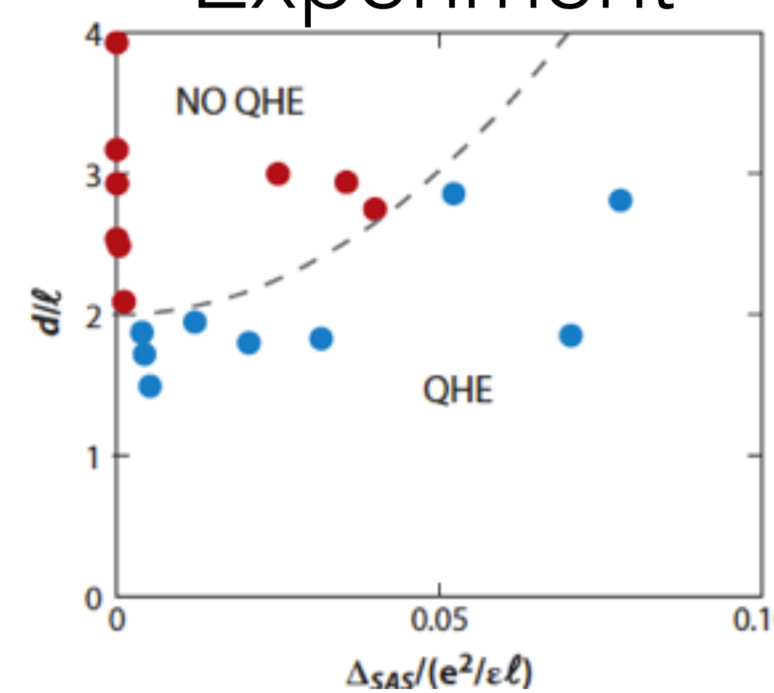
Bonesteel et al. PRL(1996)

Numerics



Möller et al. PRL (2008)

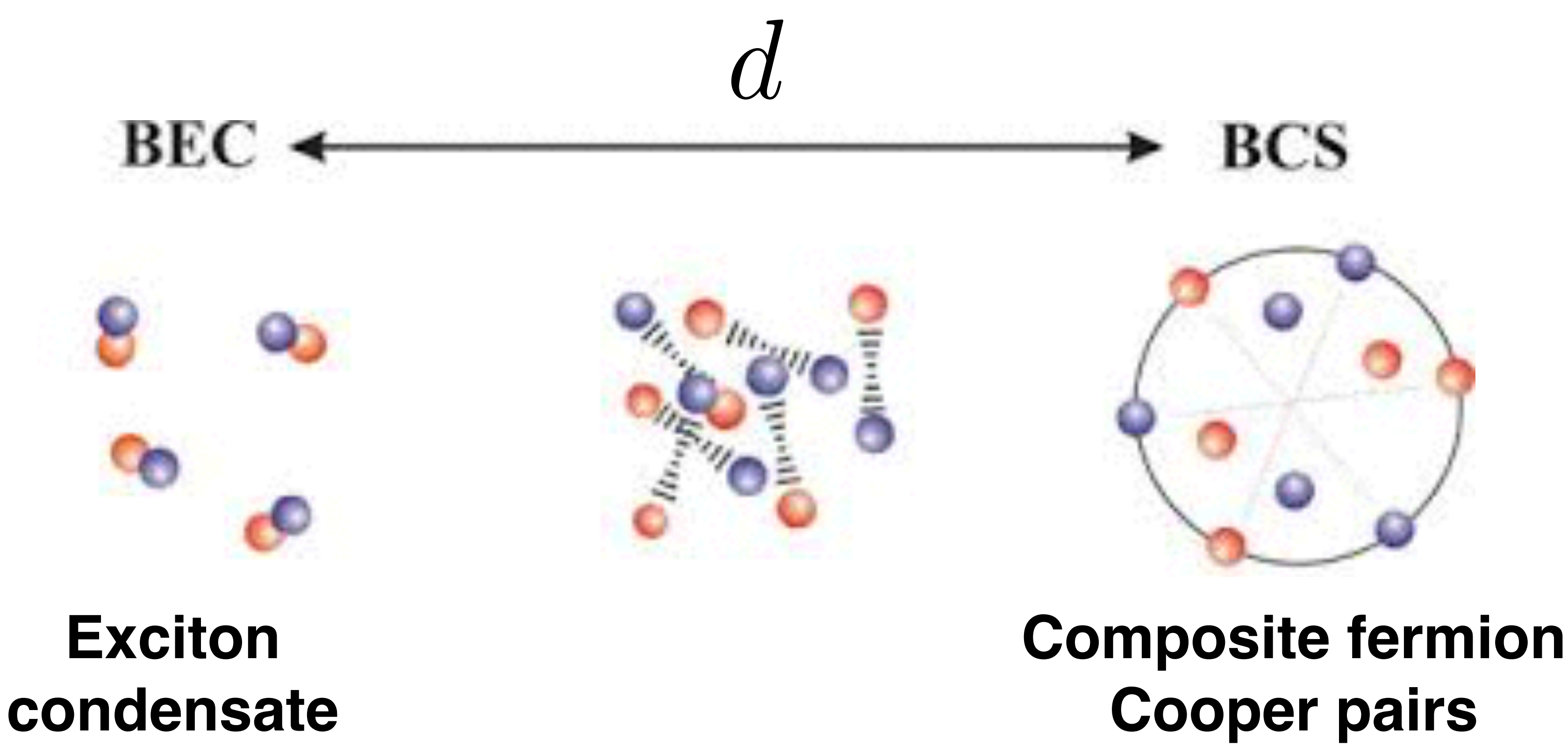
Experiment

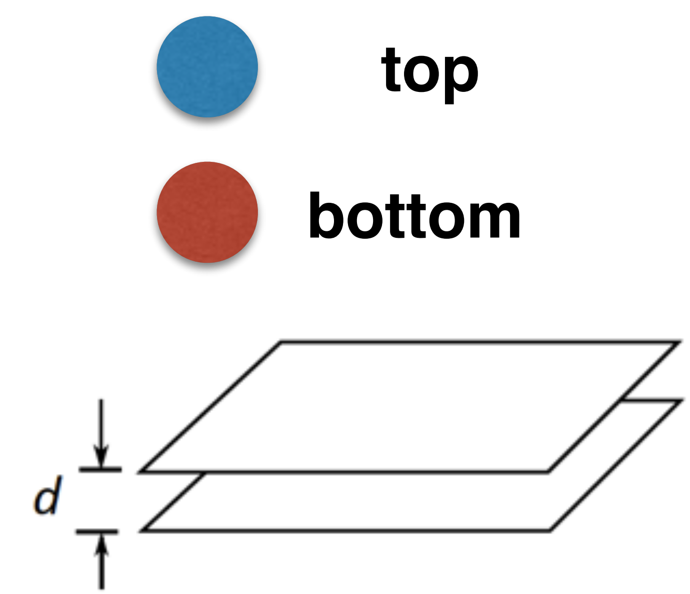


Eisenstein, ARCMP (2014)

Bilayer exciton condensate and Composite fermion metal

- A special particle-hole invariant "cooper pairing" of composite fermions is equivalent to exciton condensate:

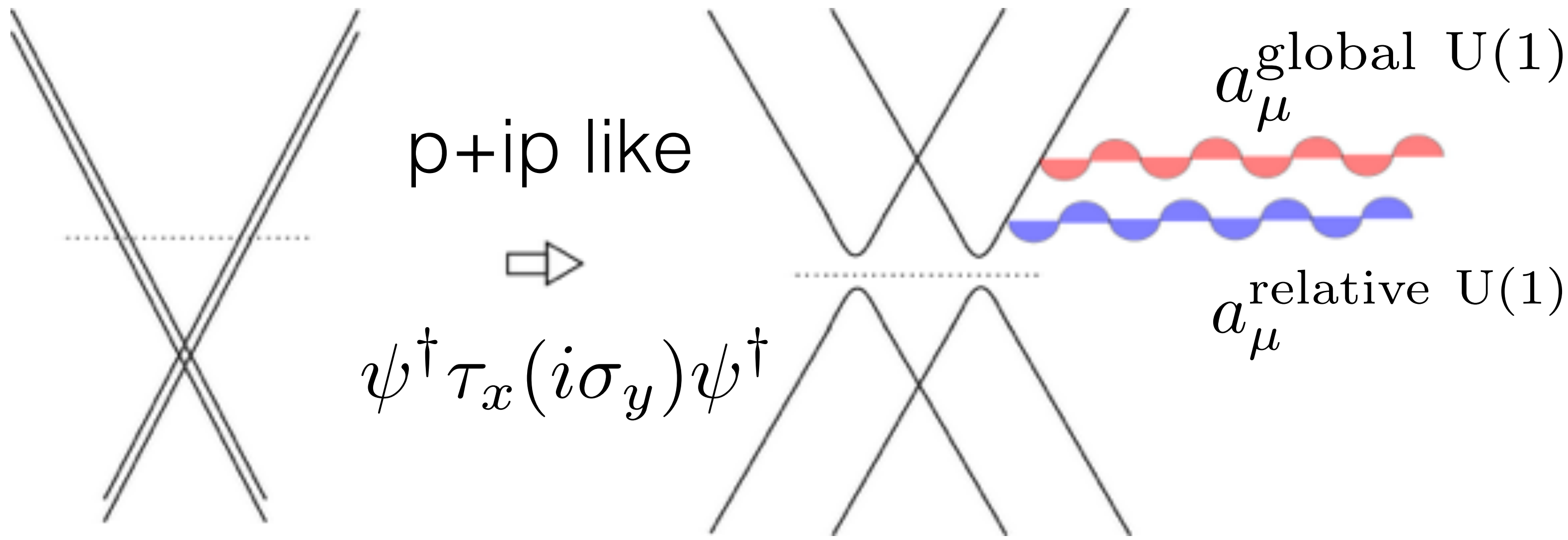




$$\hat{\Delta} = i\psi^\dagger \sigma_y \tau_x \psi^\dagger \sim i\psi^\dagger_{\text{top}} \sigma_y \psi^\dagger_{\text{bottom}}$$

I. Sodemann, I. Kimchi, C. Wang, T. Senthil,
Phys. Rev. B **95**, 085135 (2017).

Exciton condensate from CF pairing



$$a_+ = \frac{a_1 + a_2}{2}$$

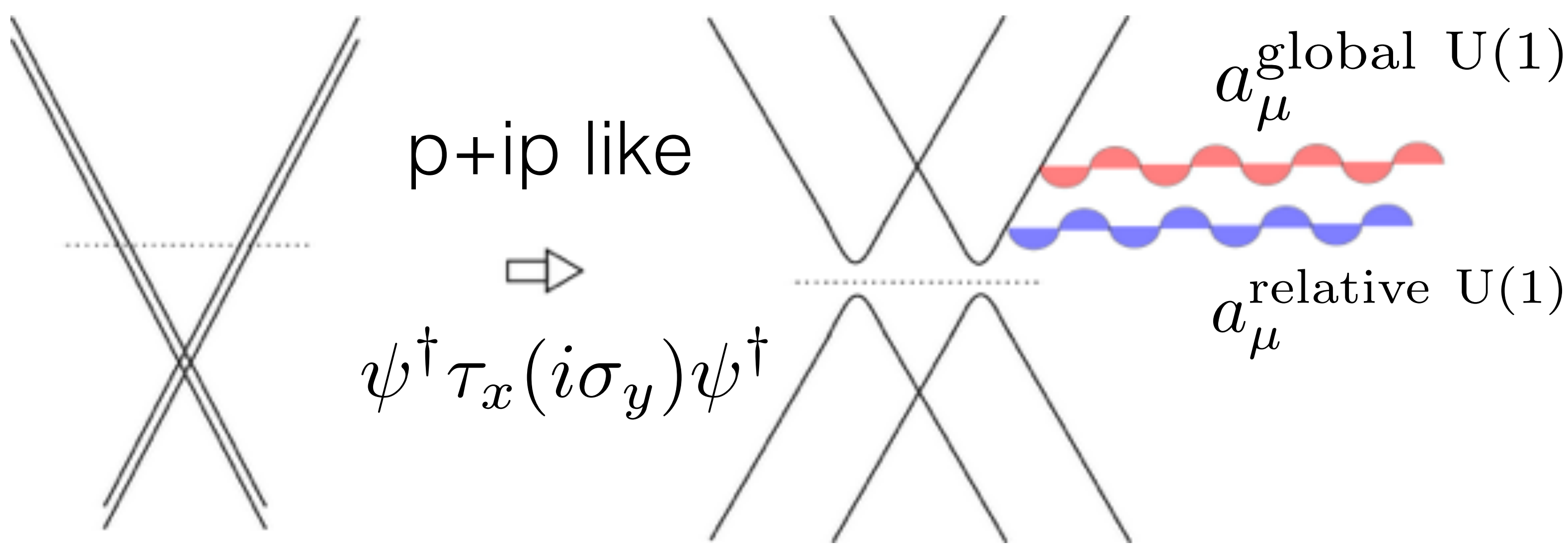
$$a_- = \frac{a_1 - a_2}{2}$$

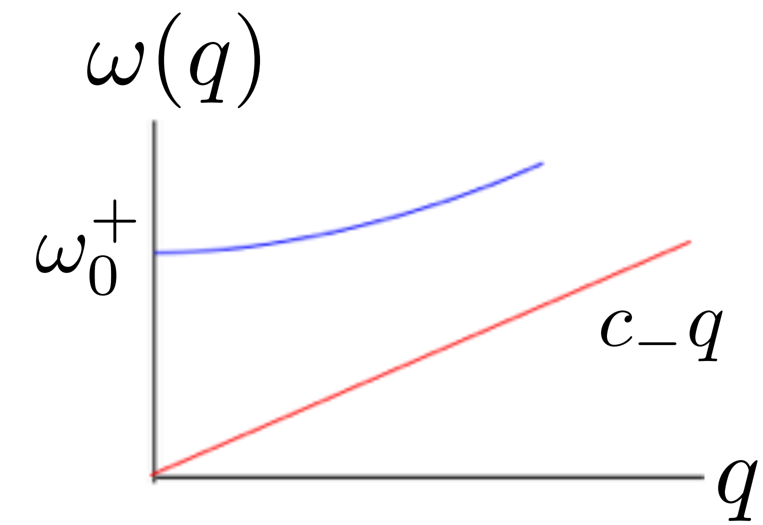
- Symmetric gauge field is gapped via Higgs.
- Anti-symmetric gauge field remains gapless. 2+1 Maxwell theory has a spontaneously broken symmetry:

$$\langle \mathcal{M}_-(r)\mathcal{M}_-^\dagger(0)\rangle \xrightarrow{|r|\to\infty} \text{const} \qquad n^e_{\text{top}} - n^e_{\text{bottom}} = \frac{\nabla \times \vec{a}_-}{2\pi}$$

$\Rightarrow$ $\langle c^\dagger_{\text{bottom}} c_{\text{top}} \rangle \propto e^{i\phi}$ The state is an exciton condensate!

Relative u(1) photon = Goldstone mode





- Photon is exciton condensate "spin-wave".
- Electric charges under field a_- are vortices of condensate order parameter:

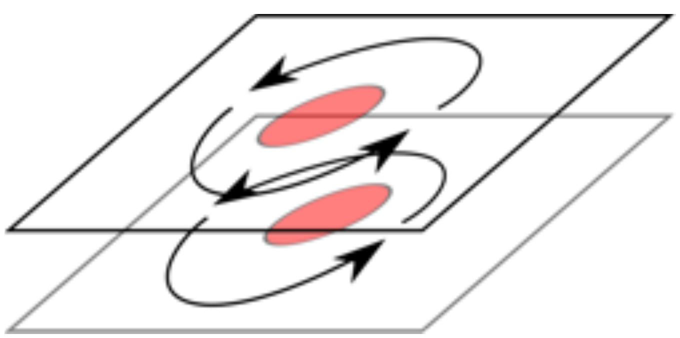
$$4\pi q_- \leftrightarrow vorticity$$

$$\hat{z} \times (\vec{j}_{\text{top}}(r) - \vec{j}_{\text{bottom}}(r)) = \frac{\nabla a_0 + \partial_t \vec{a}}{4\pi}$$

Abrikosov vortices = merons

- Abrikosov vortices carry half charge:

$\pi - \text{vortex}$



$Q = 1/2$

$$n^e_{\text{top}} + n^e_{\text{bottom}} = \frac{\nabla \times \vec{a}_+}{2\pi} \quad \Rightarrow \quad Q_\pi = \pm\frac{1}{2}$$

- Abrikosov vortices have a complex fermion zero mode:

Layer X-change

$|0\rangle \longrightarrow |1\rangle$

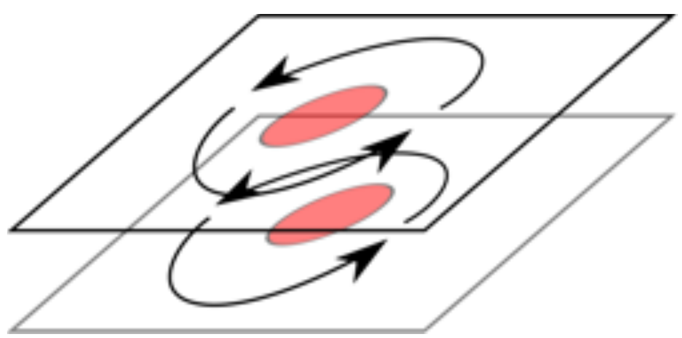
$|1\rangle \equiv \psi_0^\dagger |0\rangle \longrightarrow |0\rangle$

	q_-	(vorticity)
$\|0\rangle$	$1/2$	2π
$\|1\rangle$	$-1/2$	-2π

Abrikosov vortices = merons

- Two π Abrikosov vortices of opposite vorticity are mutual semions

$\pi - \mathrm{vortex}$



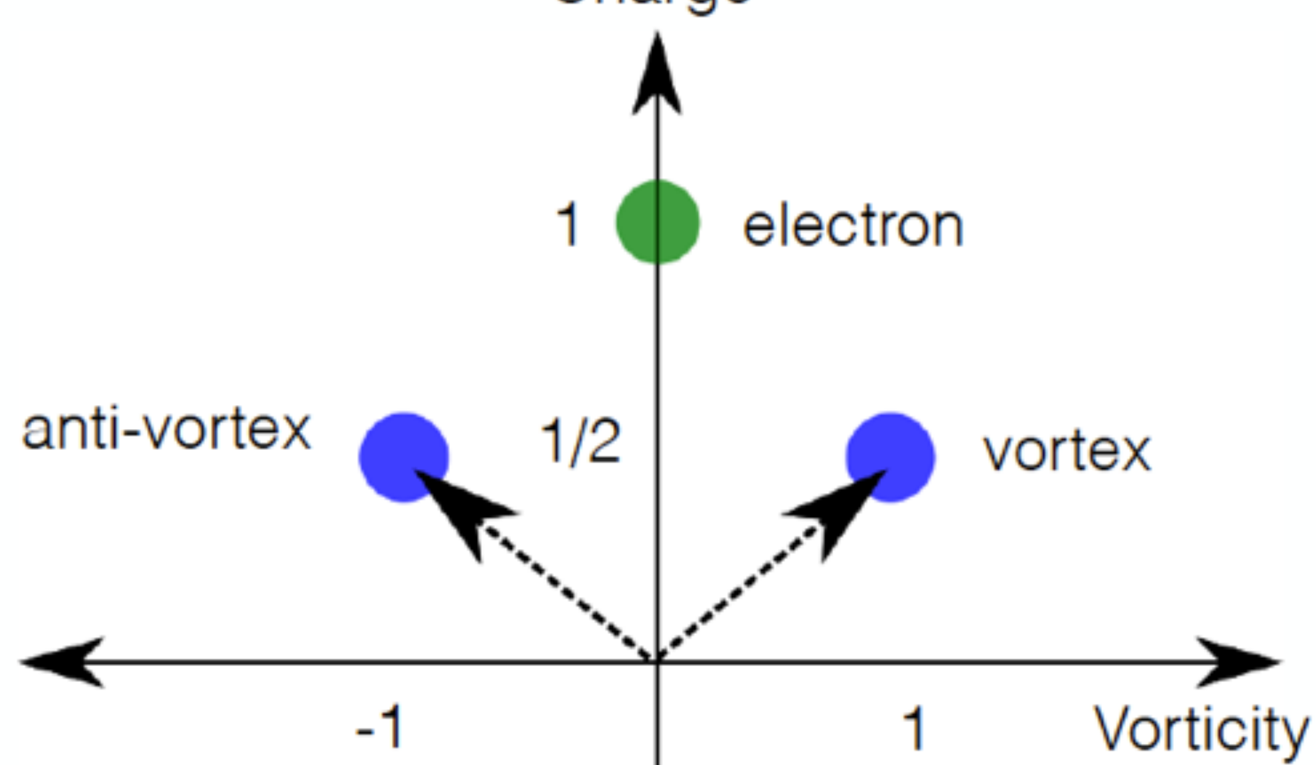
$Q = 1/2$

$$|0\rangle$$

$$|1\rangle \equiv \psi_0^\dagger |0\rangle$$

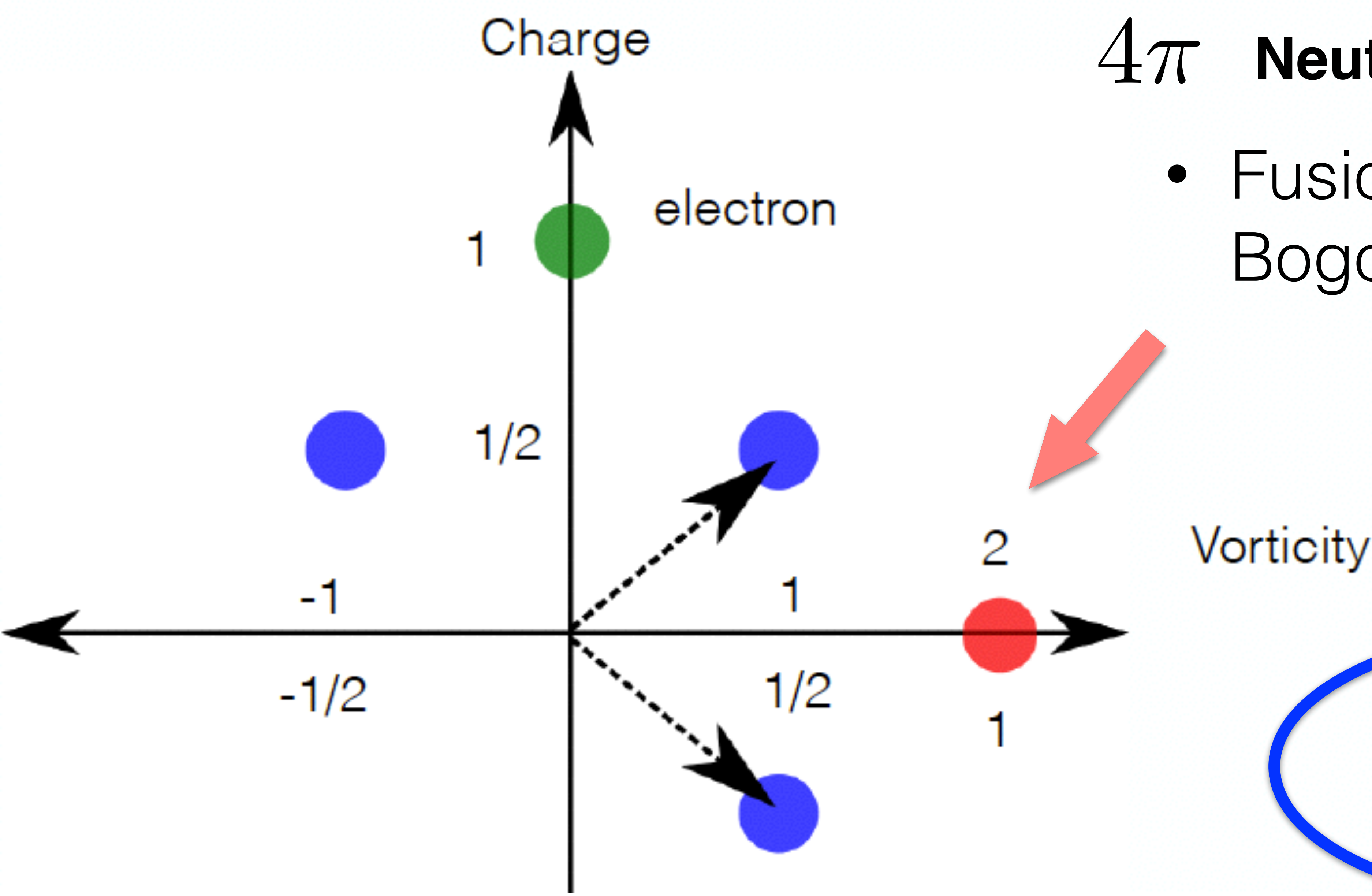
- Their fusion is a fermion:

The electron (with layer charge imbalance neutralized by condensate).



Bogoliubov fermion

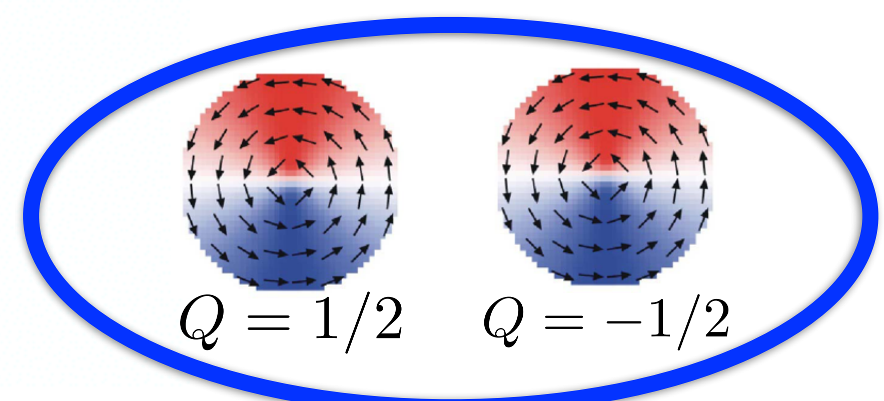
- Consider fusing two Abrikosov vortices of opposite flux but same a_- charge (order parameter vorticity):



4π **Neutral Vortex**

- Fusion is Bogoliubov fermion

4π

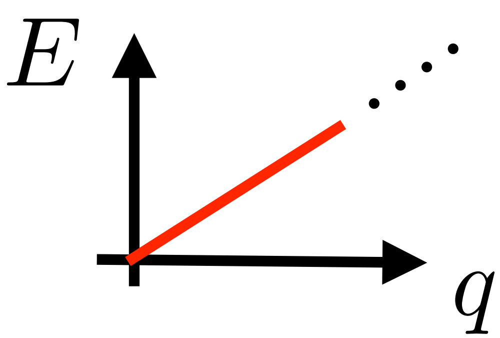


Dictionary

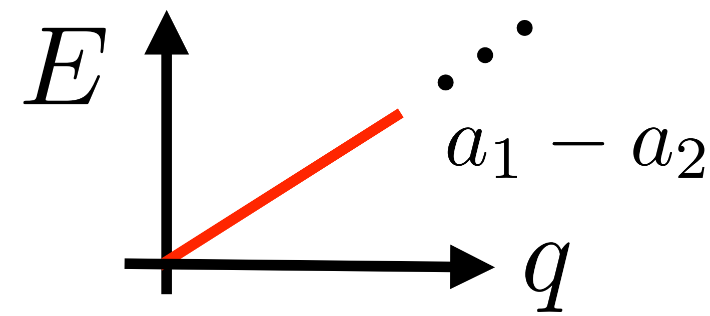
Exciton condensate

Composite fermion superconductor

Spin-wave



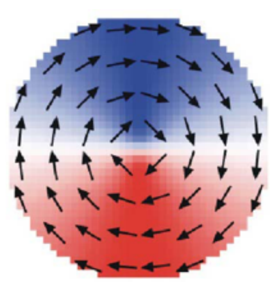
Photon



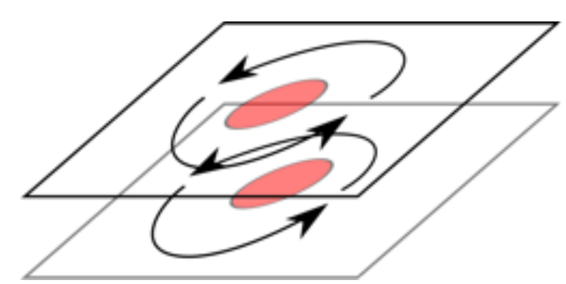
XY vortex

$Q = 1/2$

2π winding



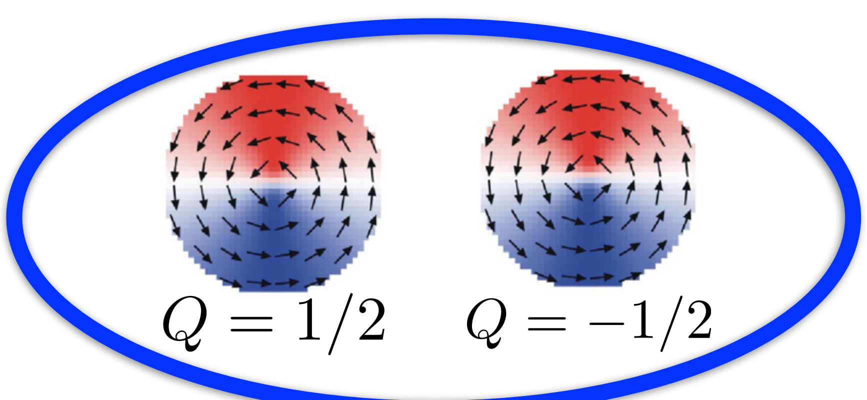
Abrikosov vortex



π flux

$Q = 1/2$

4π **neutral vortex**



Composite fermion

Charge neutral

Dipole carrying

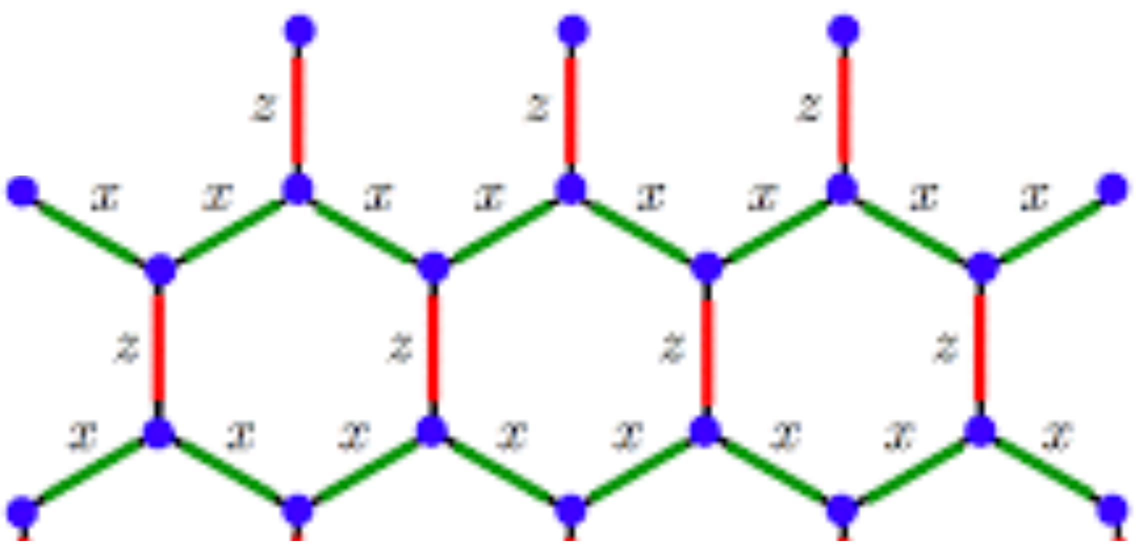
Fractionalization w/out magnetic fields

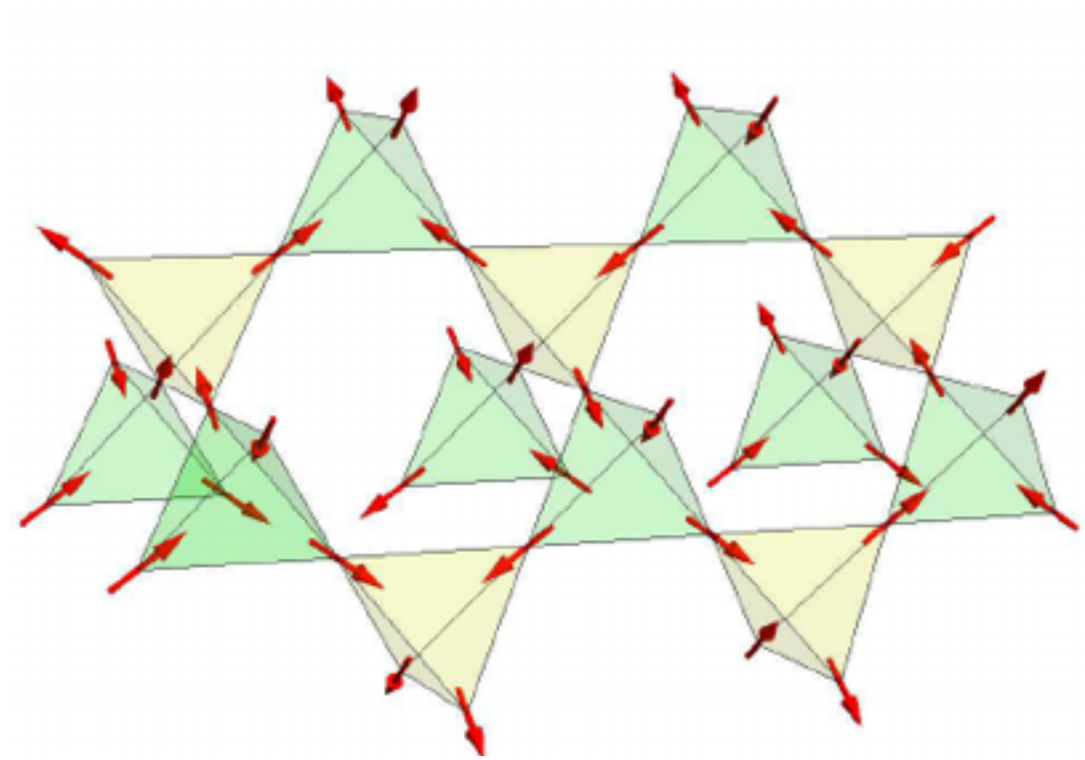

Anderson


Kitaev


Laughlin

- Spin liquids in frustrated magnets.

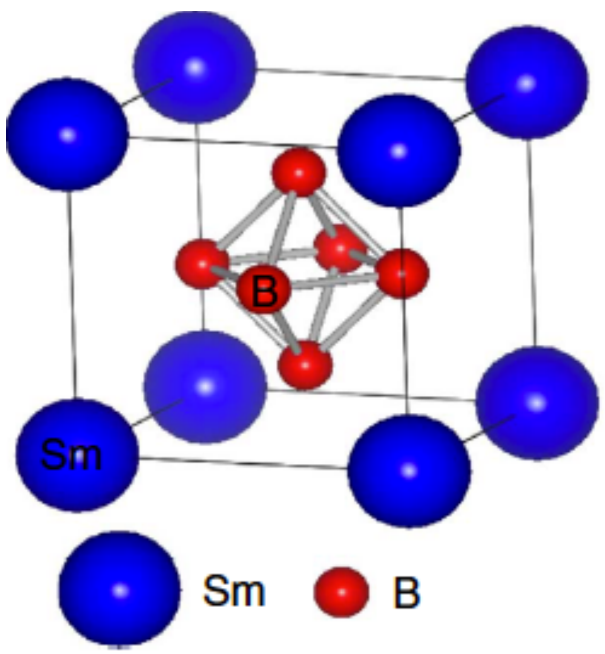




- Bosonic Laughlin state can be viewed as chiral spin liquid after mapping bosons to spins.
- “Smoking gun” experimental signatures?
- Fractionalization beyond the realm of frustrated magnets or quantum Hall?

Puzzles of SmB_6

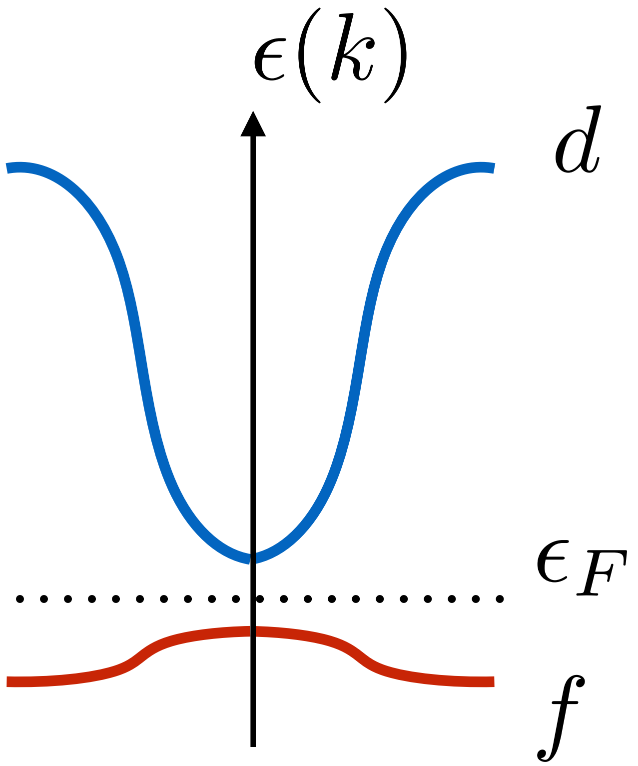
- Simple cubic structure.
- All action happens in Samarium.
- Traditional picture of mixed valence insulator:



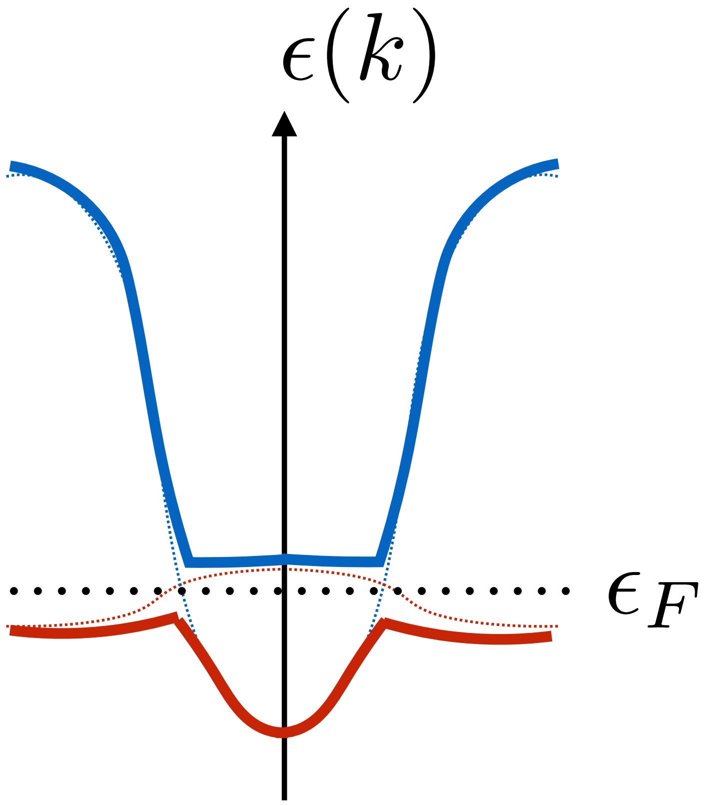
$$[\mathrm{Xe}]4f^6 5d^0 6s^2$$

$$5d^1 + 4f^5 \leftrightharpoons 4f^6$$

Atomic limit



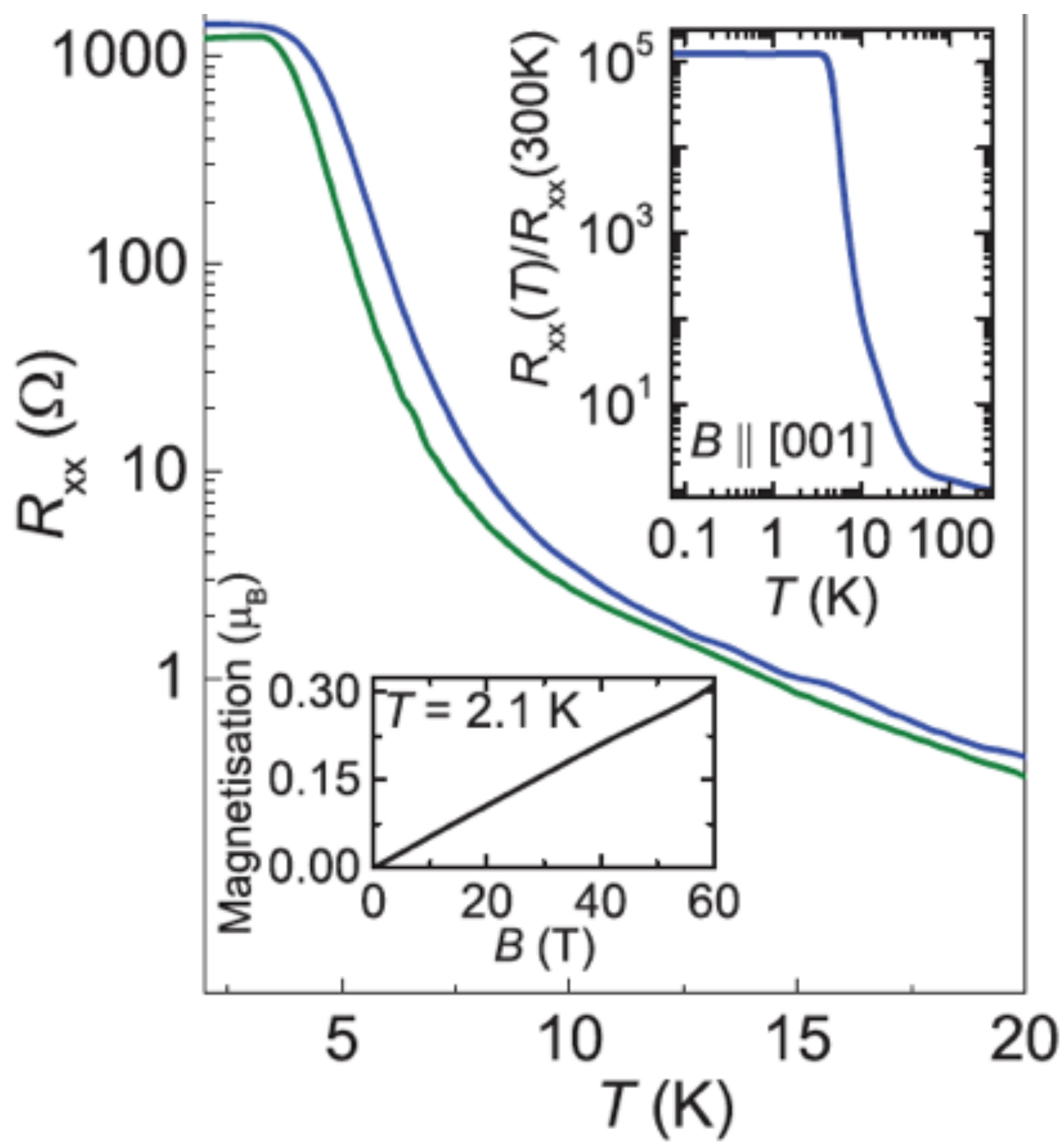
Mixed valence



SmB_6 puzzling behavior

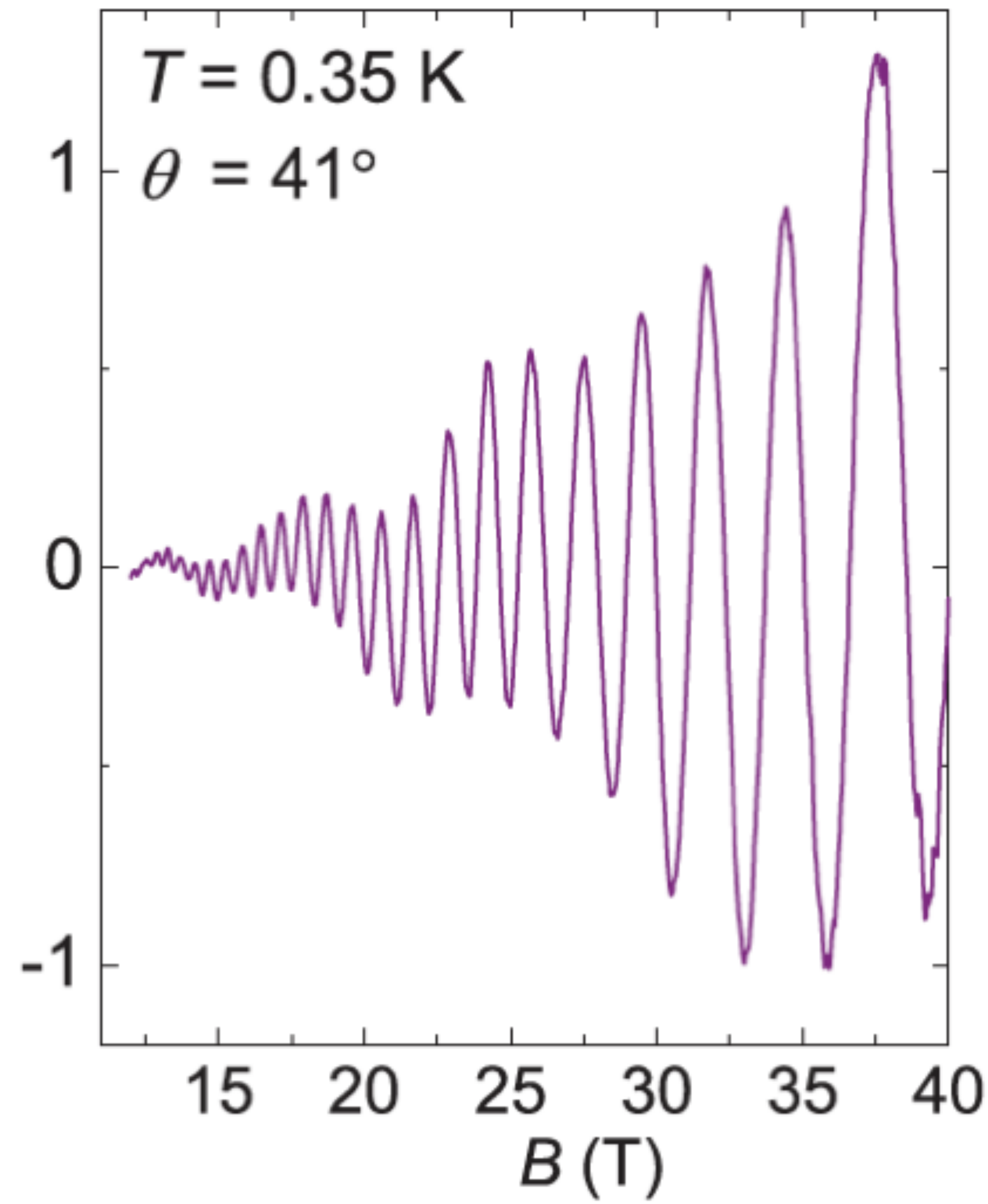
- Insulating behavior from charge transport:

$$\rho \approx \rho_0 e^{\frac{\Delta}{T}} \quad \Delta \approx 10 meV$$



B. S. Tan et al., Science (2015).

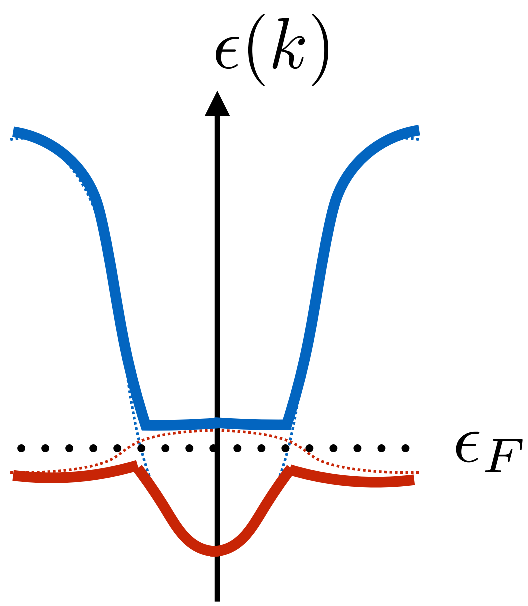
- De Haas-van Alphen effect visible at $B \sim 5T$



SmB_6 puzzles

- Could be magnetic breakdown?

Zhang, Song, Wang, PRL (2016).
Knolle and Cooper, PRL (2015).



Gap:

$$\Delta \sim 10meV$$

Cyclotron:

$$\omega_c \approx 0.2meV \;\; B[T]$$

Theory oscillations visible at $B \sim 50T$

Experiment oscillations visible at $B \sim 5T$

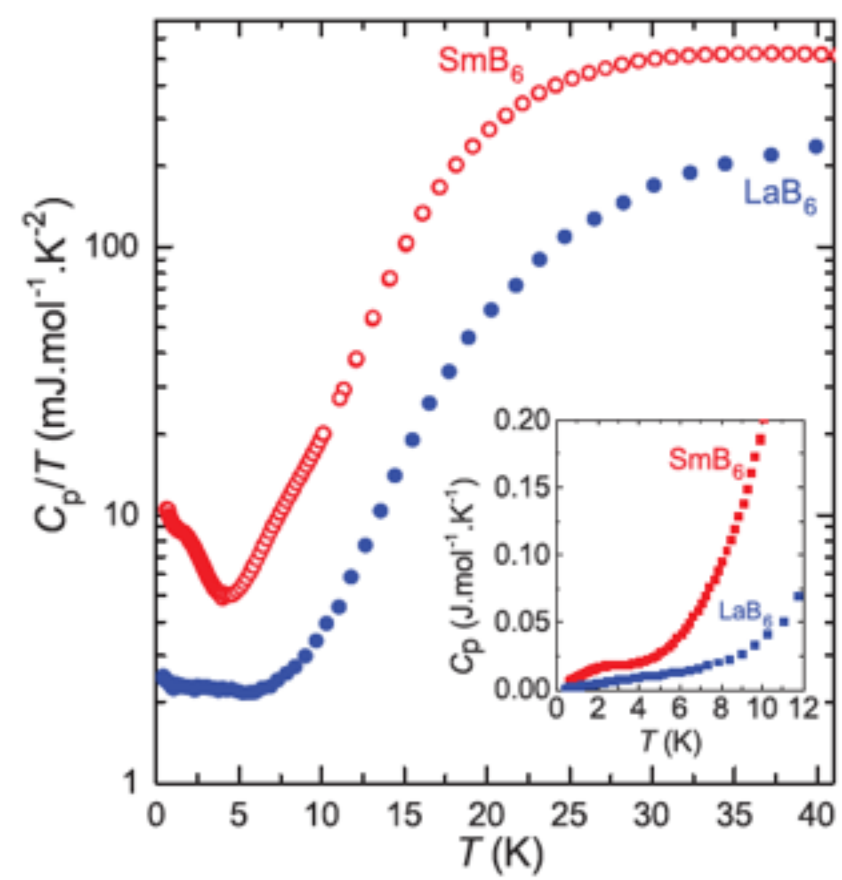
- Other anomalies:

Specific heat to temperature ratio has finite intercept:

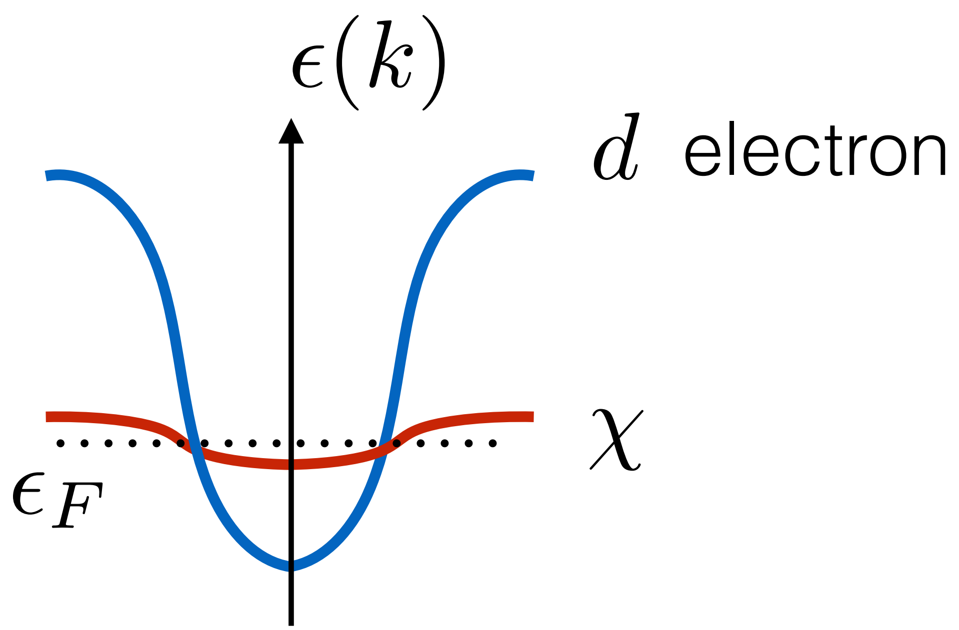
$$\gamma = \frac{C}{T}$$

Like in a fermi sea

$$C_{\text{fermions}} \propto \gamma T \qquad C_{\text{phonon}} \propto T^3$$



“Composite exciton Fermi liquid”



One option:
bosons condense

$$\langle b \rangle \neq 0$$

=> Metal (“boring”)

$$N^d_{electrons} = N^b = N^\chi$$

Fermi-bose mixture:

$b^\dagger$: spinless boson

$\chi^\dagger_\sigma$: neutral spinfull fermion

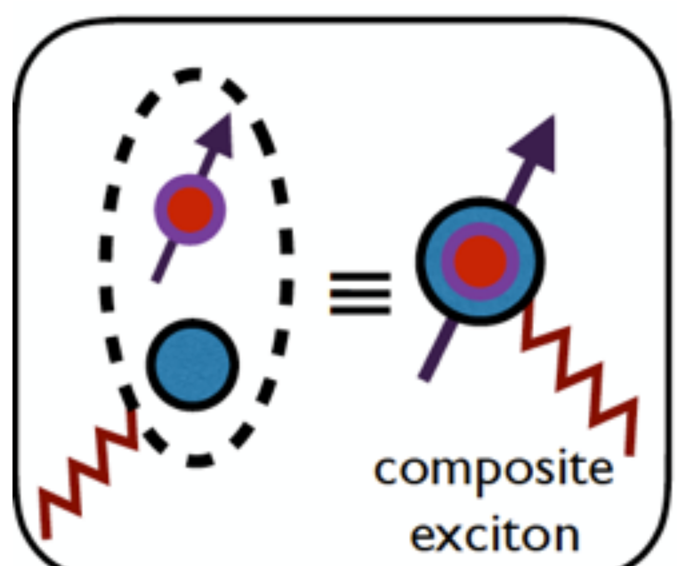
$d^\dagger_\sigma$: d-electron

More “interesting” option:

Bosons bind with d electrons

b and d attract:

$$-U_{df} \sum_i n_i^f n_i^d$$



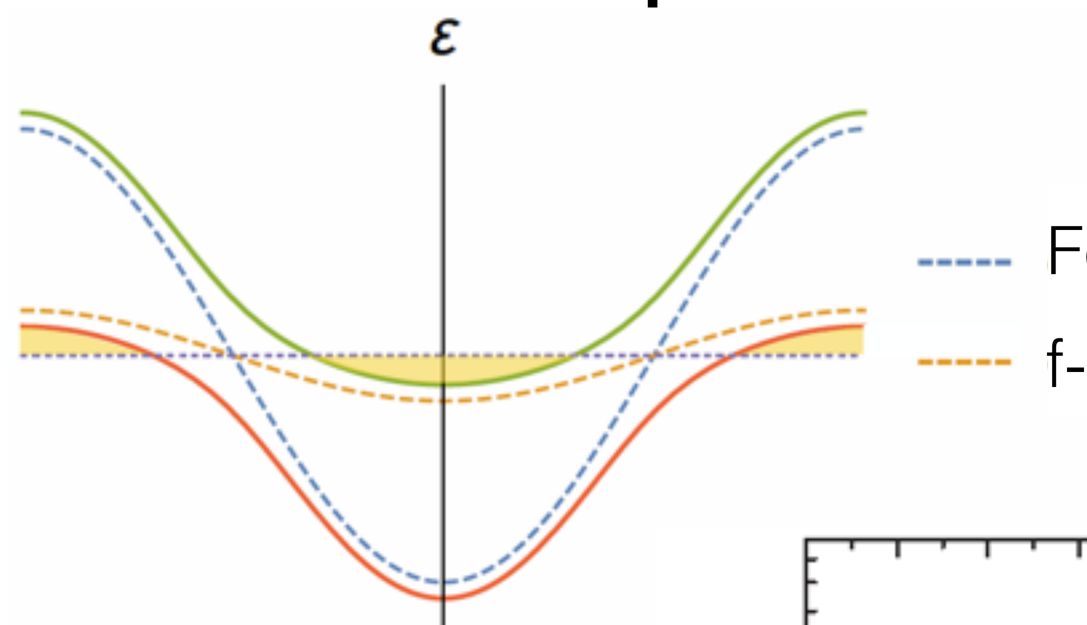
Composite fermionic exciton:

$$\psi_{\mathbf{k}\alpha} \equiv b\, d_{\mathbf{k}\alpha}, \quad \psi^\dagger_{\mathbf{k}\alpha} \equiv b^*\, d^\dagger_{\mathbf{k}\alpha}$$

Bound state of “f-holon"
and d electron.

Properties of “Composite exciton Fermi liquid”

Fractionalized
fermi sea with two pockets
(“semi-metal”)



Some properties:

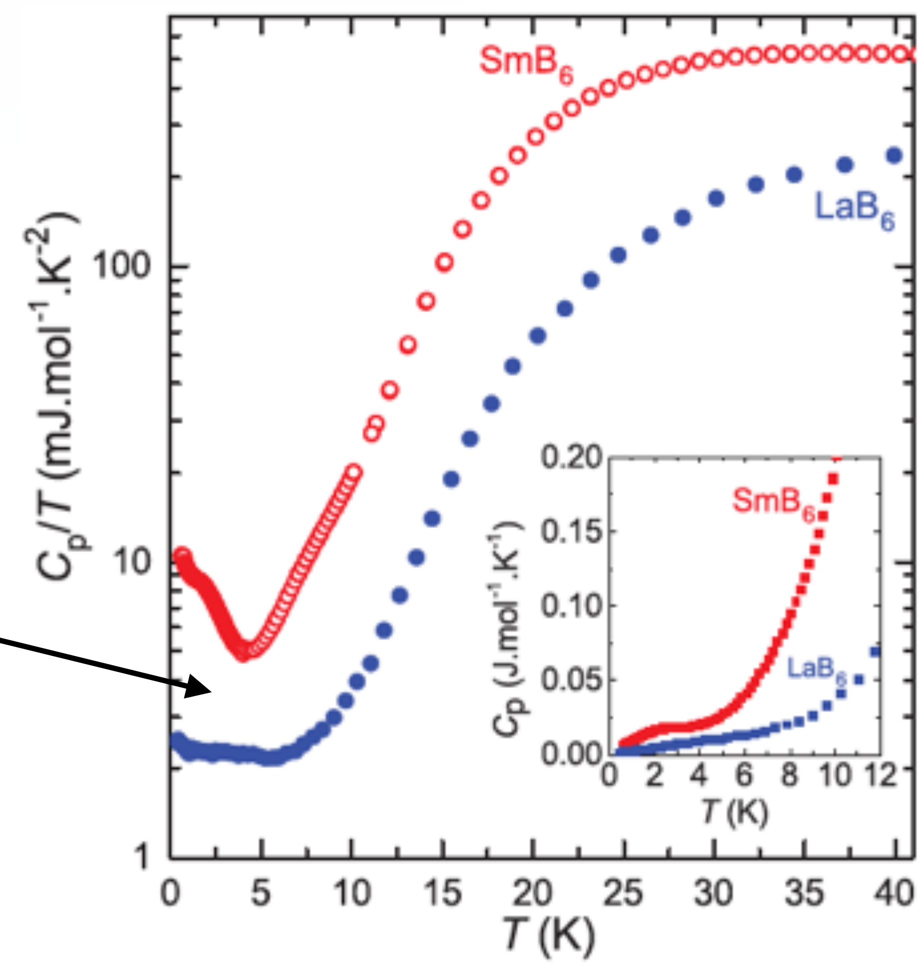
- Essentially linear specific heat:

$$C = \gamma T \qquad \gamma \sim \ln(1/T)$$

- Sub-gap optical conductivity:

$$\mathrm{Re}[\sigma(\omega)] = \omega^2\left(\frac{\epsilon_b - 1}{4\pi}\right)^2 \frac{1}{\mathrm{Re}[\sigma_{ce}(\omega)]}$$

Upturn might indicate other physics at lower temperature



B. S. Tan et al., Science (2015).

D. Chowdhury, I. Sodemann, T. Senthil, arXiv:1706.00418 (2017).
I. Sodemann, D. Chowdhury, T. Senthil, arXiv:1708.06354 (2017).

The end of the beginning!

Conceptual frontier:

- Topological matter beyond free fermions.
- Fractionalization and topology in 3D.
- Gapless fractionalized phases in 2D and 3D.
- Novel non-pertubative approaches to interacting systems.

Real world frontier:

- New probes for fractionalized matter.
- Fractionalization beyond quantum Hall and frustrated magnets.
- More cross talk between materials and models.

Non-equilibrium and transport frontier:

- Transport in fractionalized and topological matter.
- Collective behavior and broken symmetries in topological and fractionalized matter.
- Dynamics of nearly conserved quantities (hydrodynamics).