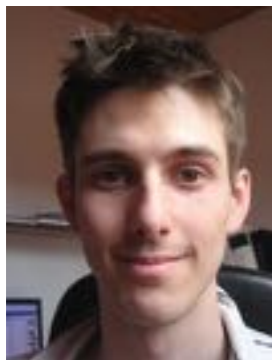


Interaction-induced transparency in the strong coupling regime of polaritons in photonic crystal waveguides

Francesco Piazza

(Max-Planck Institute for the Physics of Complex Systems)



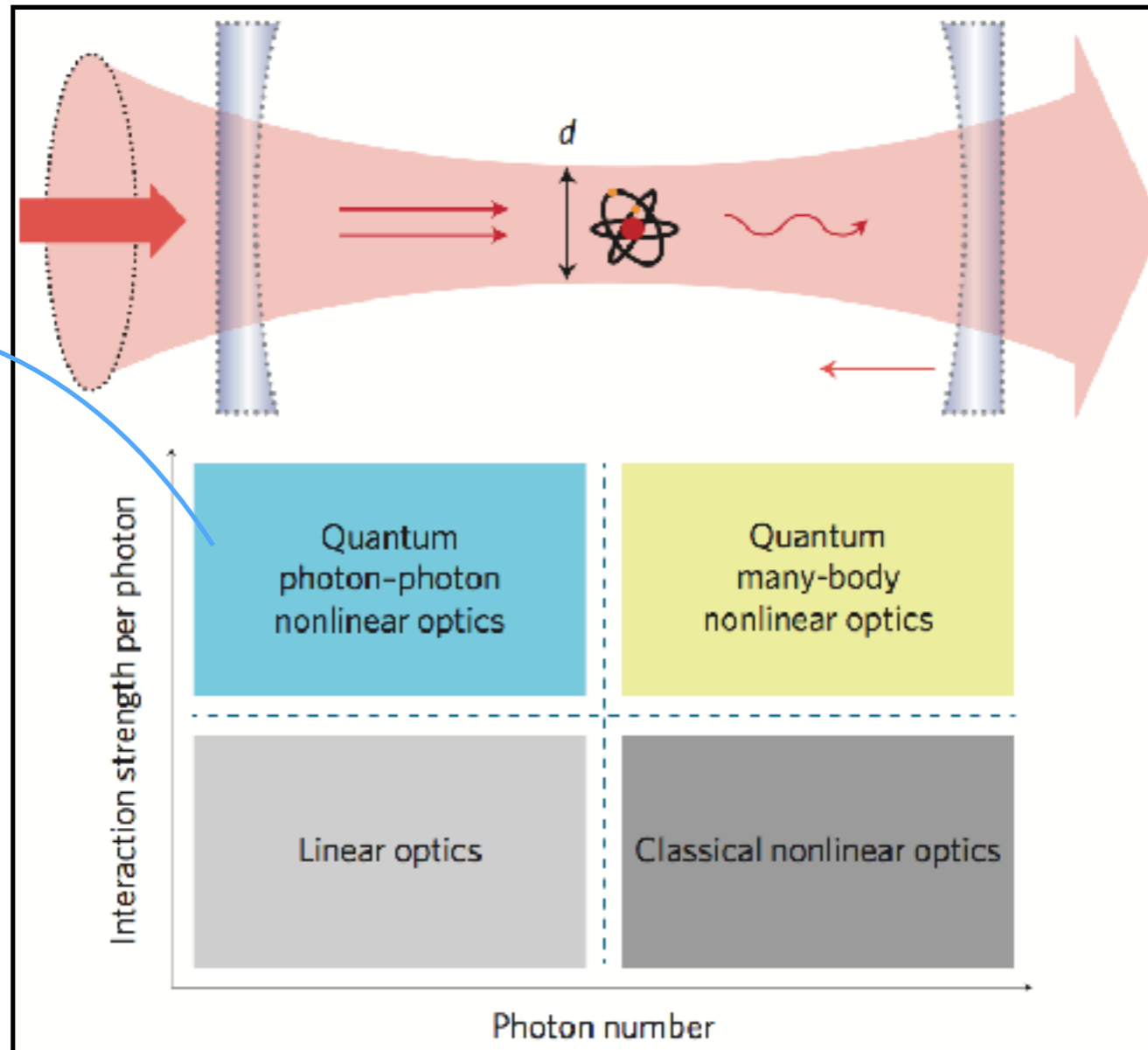
Johannes Lang
(TU Munich)



Darrick Chang
(ICFO Barcelona)

Quantum nonlinear optics

Review: Chang, Vuletic, Lukin, Nat. Phot. 8, 685 (2014).



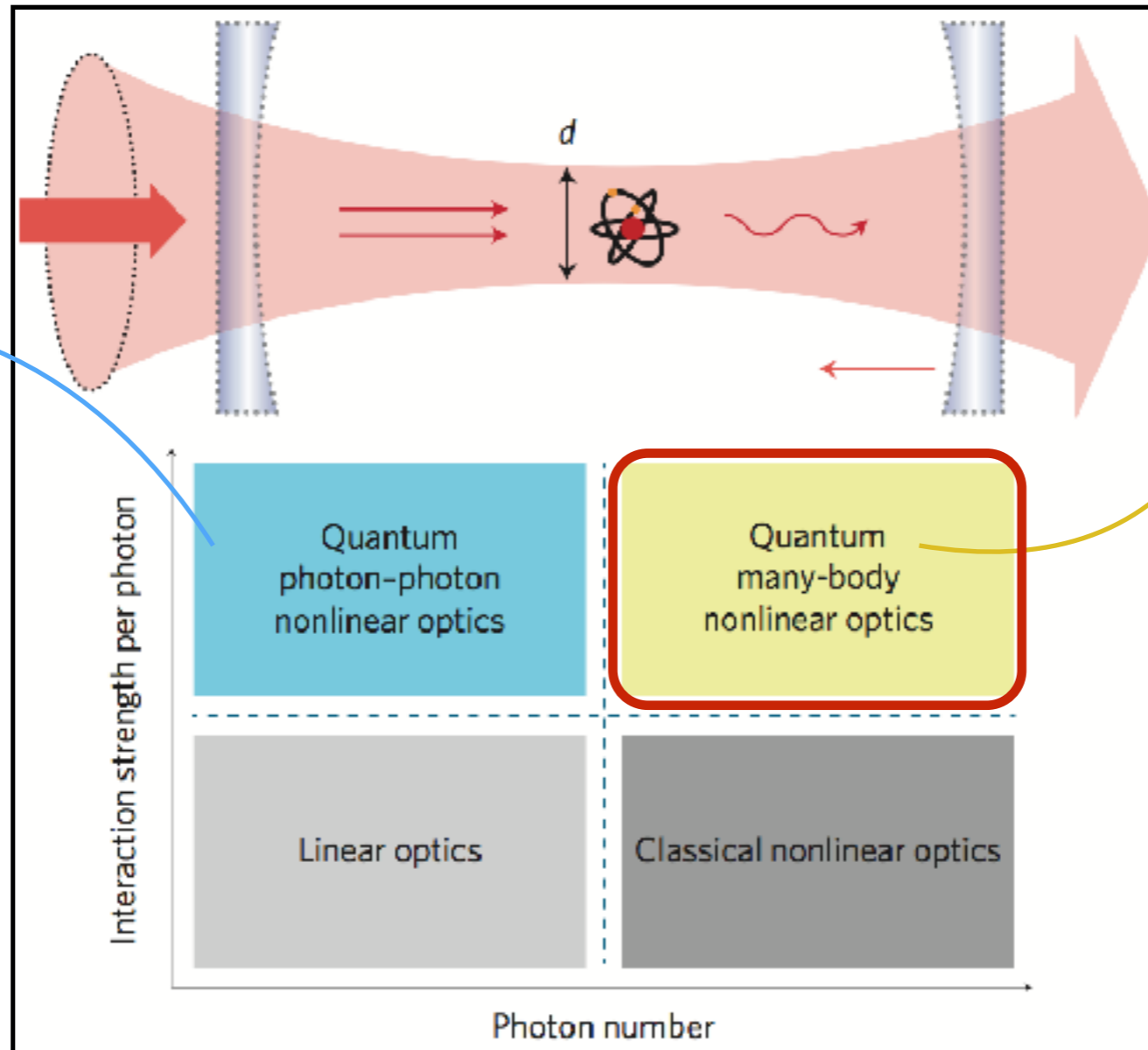
Technological motivation:
Quantum Internet

Progress has been made with several devices, coupling the electromagnetic field to:

- Trapped ions (Innsbruck)
- Molecules (Erlangen)
- Superconducting circuits (Paris, Yale, Santa Barbara, ETH, New York)
- Neutral Atoms

The challenge of quantum nonlinear optics

Review: Chang, Vuletic, Lukin, Nat. Phot. 8, 685 (2014).



Technological motivation:
Quantum Internet

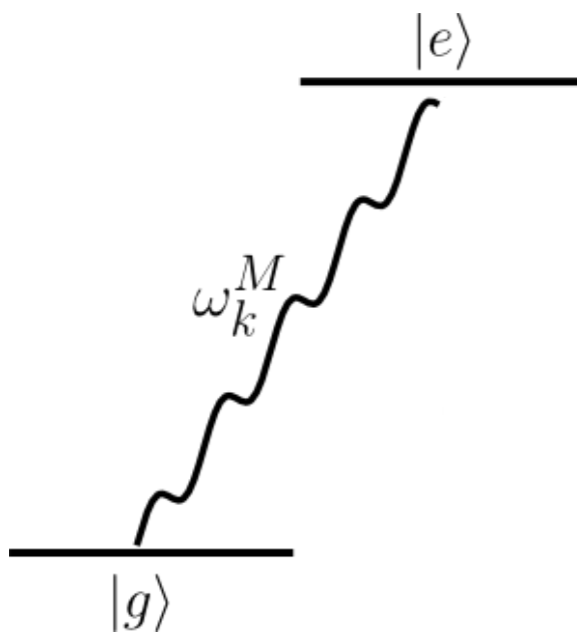
Fundamental motivation:
Novel paradigms in many-body physics

Progress has been made with several devices, coupling the electromagnetic field to:

- Trapped ions (Innsbruck)
- Molecules (Erlangen)
- Superconducting circuits (Paris, Yale, Santa Barbara, ETH, New York)
- **Neutral Atoms**

Electromagnetically-Induced Transparency (EIT)

Increase light/atom interaction probability by
slowing down the light without absorbing it

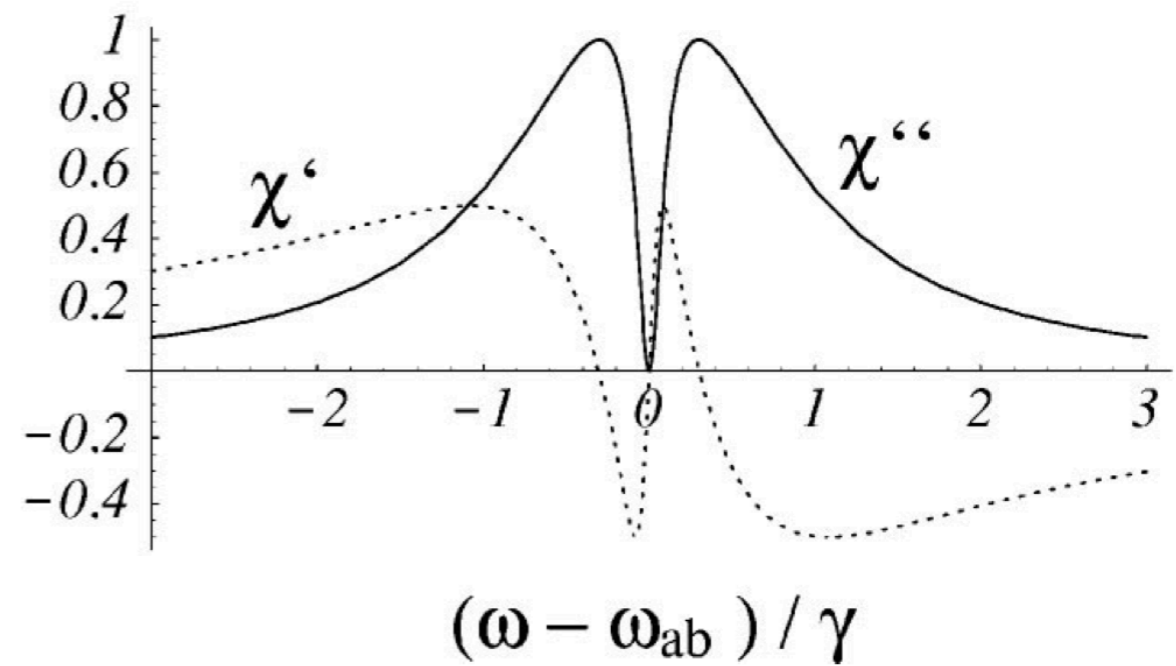
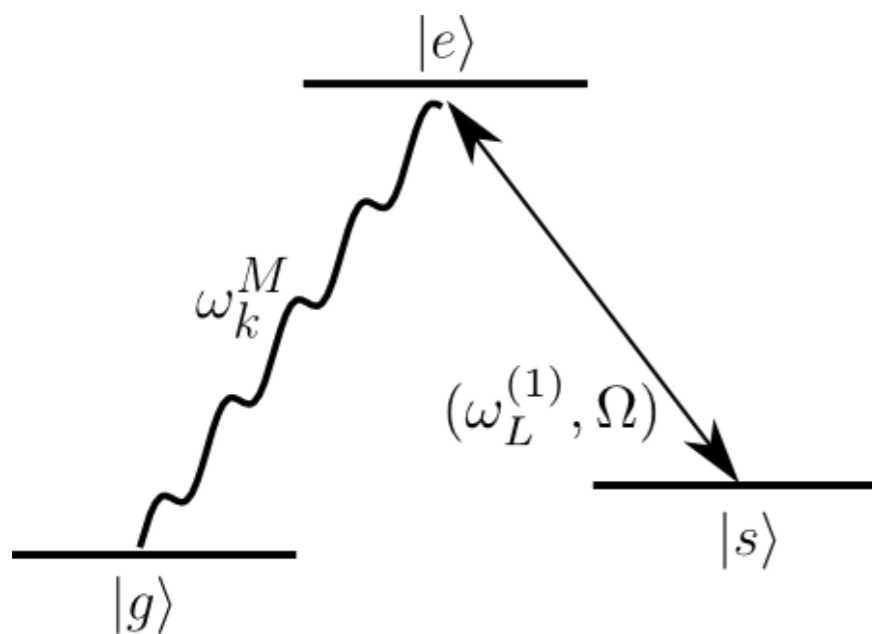


Electromagnetically-Induced Transparency (EIT)

Increase light/atom interaction probability by
slowing down the light without absorbing it

Atomic levels

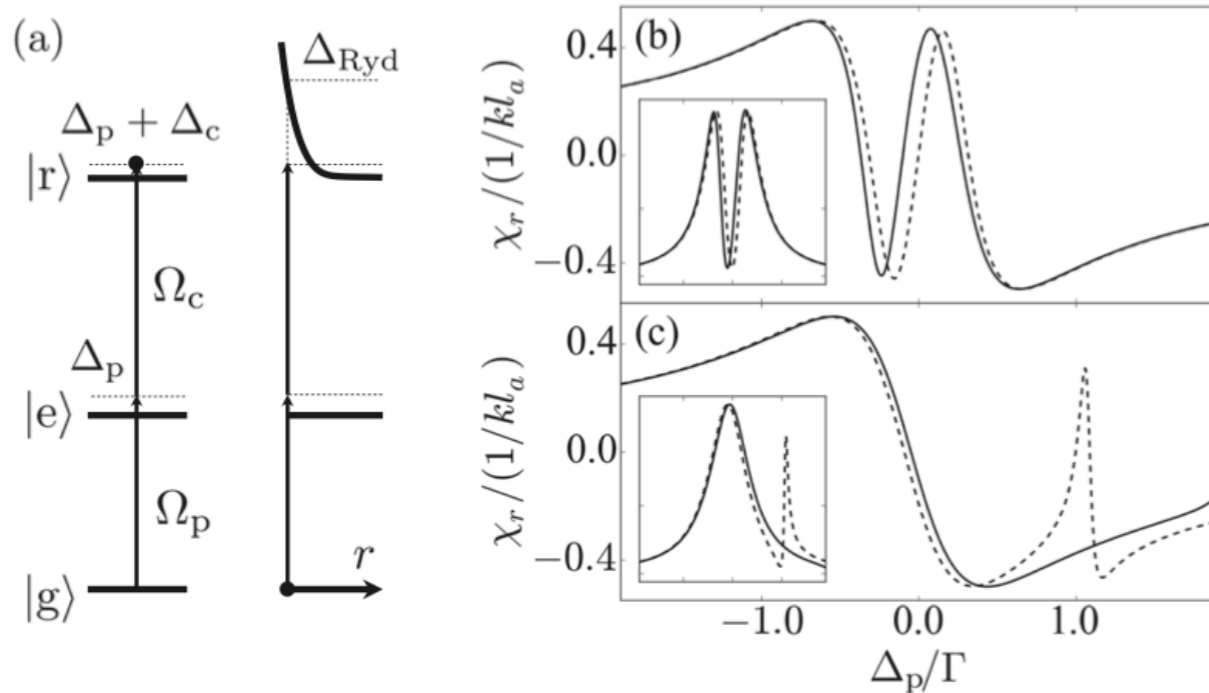
Fleischhauer, Lukin (2002)



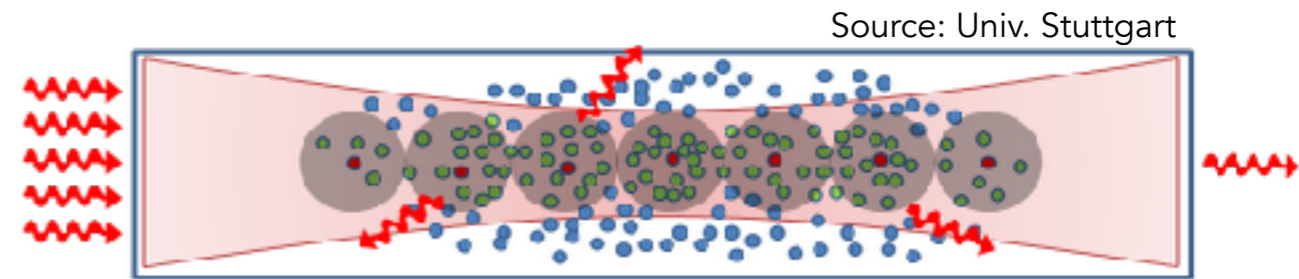
EIT with Rydberg interactions

J. Phys. B: At. Mol. Opt. Phys. 49 (2016) 152003

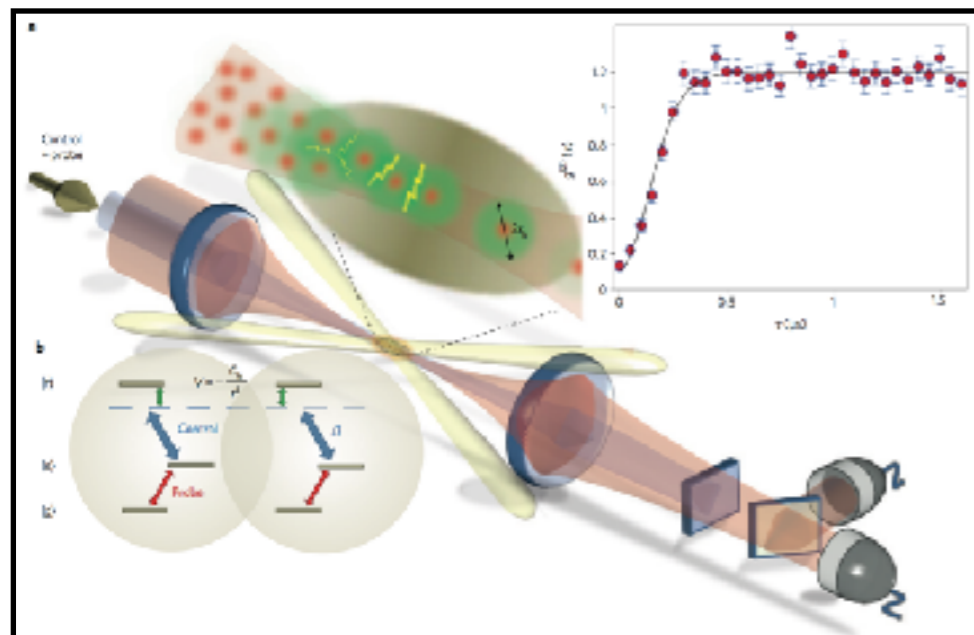
To



Rydberg atoms interact strongly over a **blockade radius**
Only one atom can be excited within radius

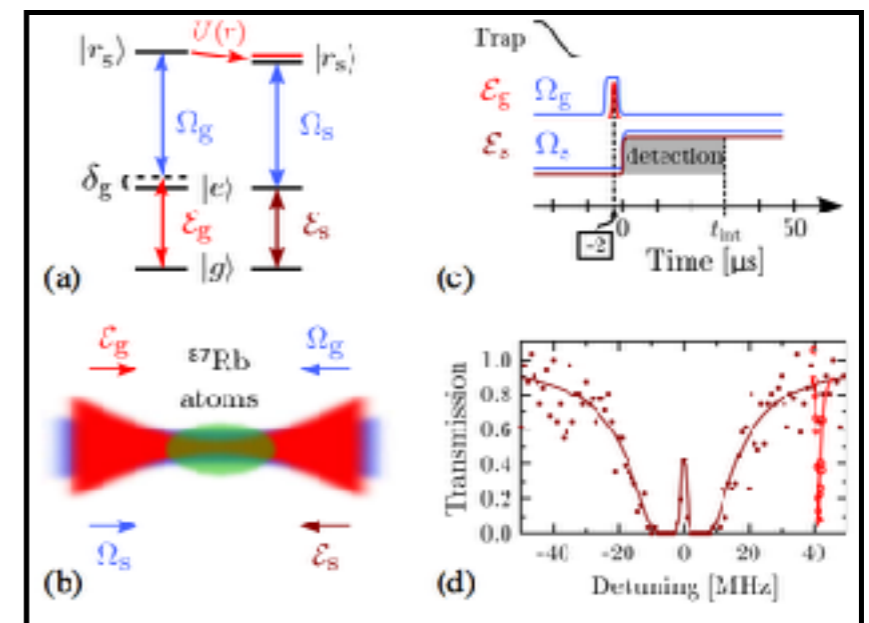


Application: Photon Blockade



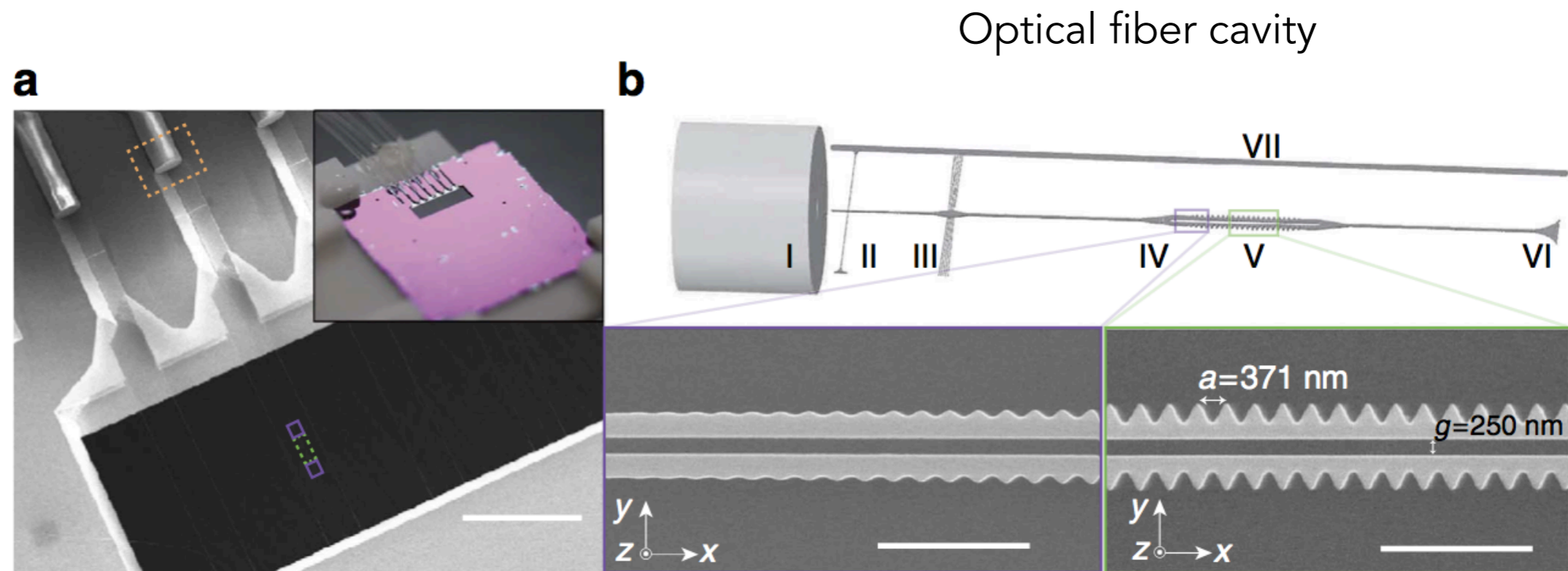
Exp.: Peyronel, et al., Nature 488, 57 (2012).

Application: All optical single-photon transistor

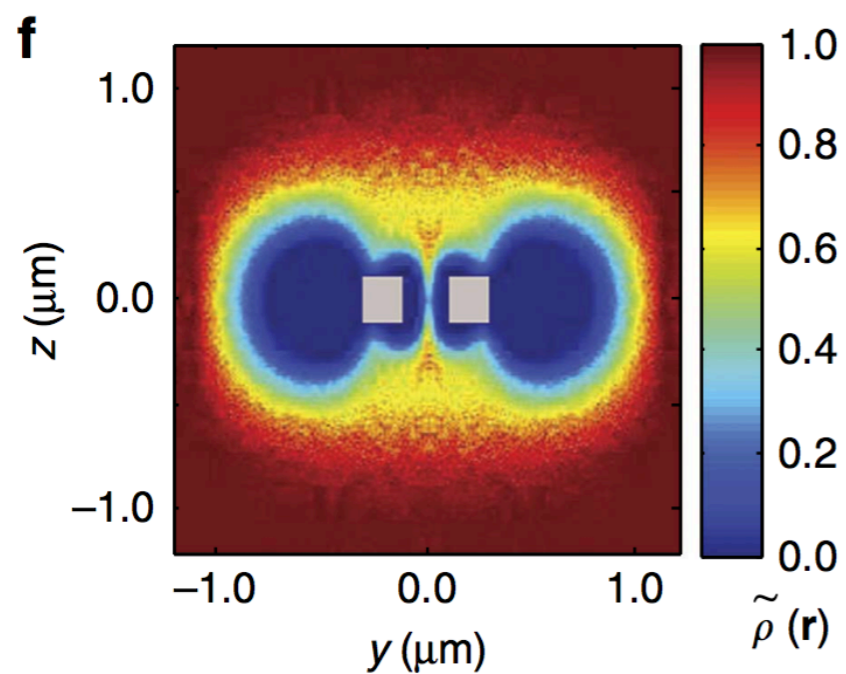


Exp.: Gorniaczyc, et al., Phys. Rev. Lett. 113, 053601

Experiment: atoms coupled to photonic crystal waveguides

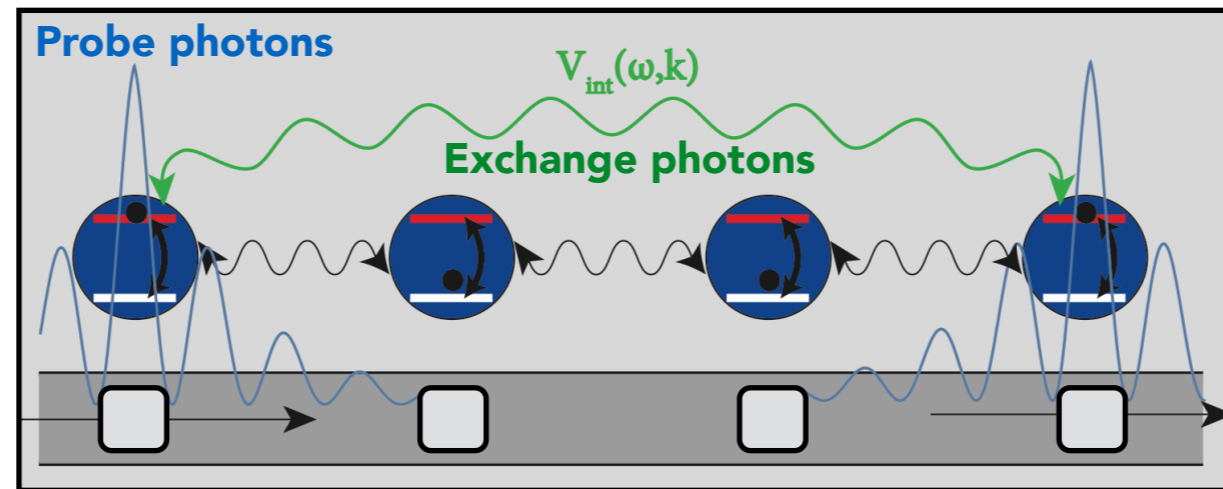


Goban et al., Nat. Comm. 5 (2014)



Atoms trapped using evanescent wave

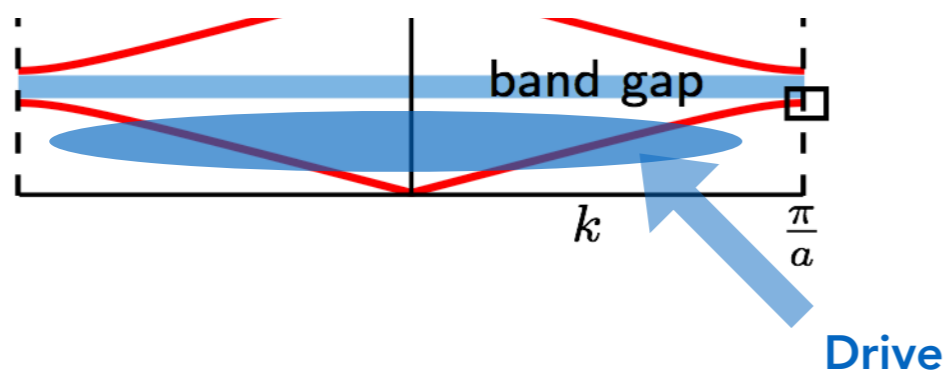
Interacting polaritons in photonic crystal waveguides



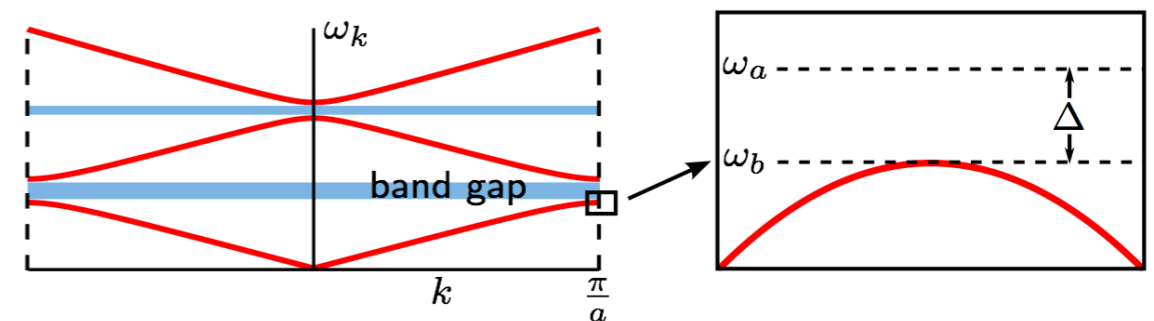
Hybrid atom-photon architecture

- Multi-level atoms fixed positions
- Probe photons within propagating band of lattice
- Exchange photons mediating interactions

Probe photons



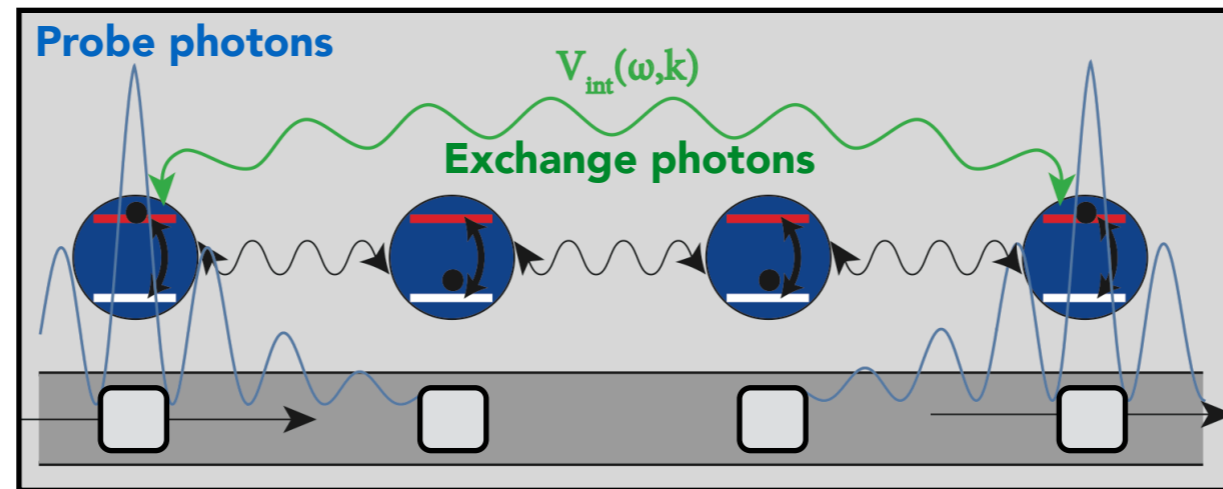
Exchange photons



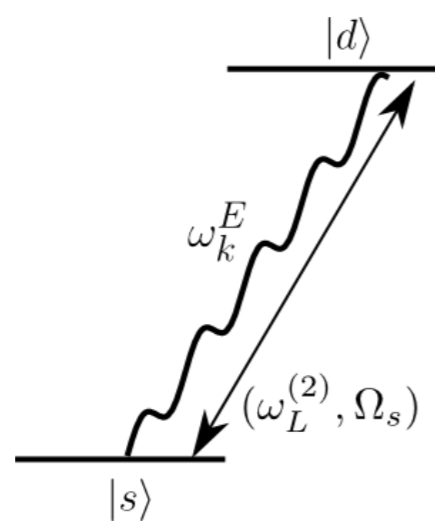
- Tune shape of interactions in space-time

J. S. Douglas, et al., Nat Photon 9, 326 (2015)

Shaping the light-mediated interactions



Atomic levels

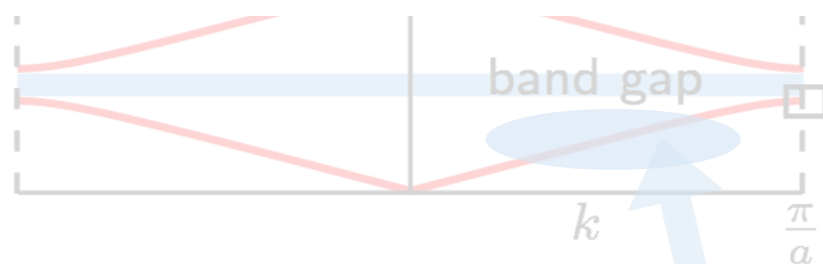


$$\hat{H}_{\text{int}} = \sum_z \left[\Omega_s e^{-i\omega_L^{(2)} t} \hat{a}_d^\dagger(z) \hat{a}_s(z) + g_E \hat{a}_E(k) e^{ikz} u_k^E(z) \hat{a}_d^\dagger(z) \hat{a}_s(z) + h.c. \right]$$

$$\hat{H}_{\text{int}}^{\text{eff}} \sim \sum_{zz'} \hat{a}_s(z)^\dagger \hat{a}_s(z) e^{-|z-z'|/L_E} \hat{a}_s^\dagger(z') \hat{a}_s(z') \quad (\text{adiabatic elimination})$$

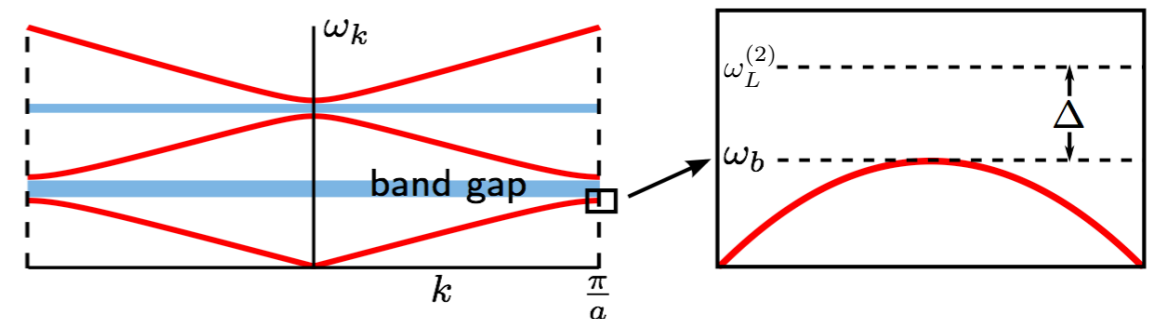
Exponential interaction range: $L_E \simeq \sqrt{\frac{\alpha_E}{\Delta}}$

Probe photons



Drive

Exchange photons



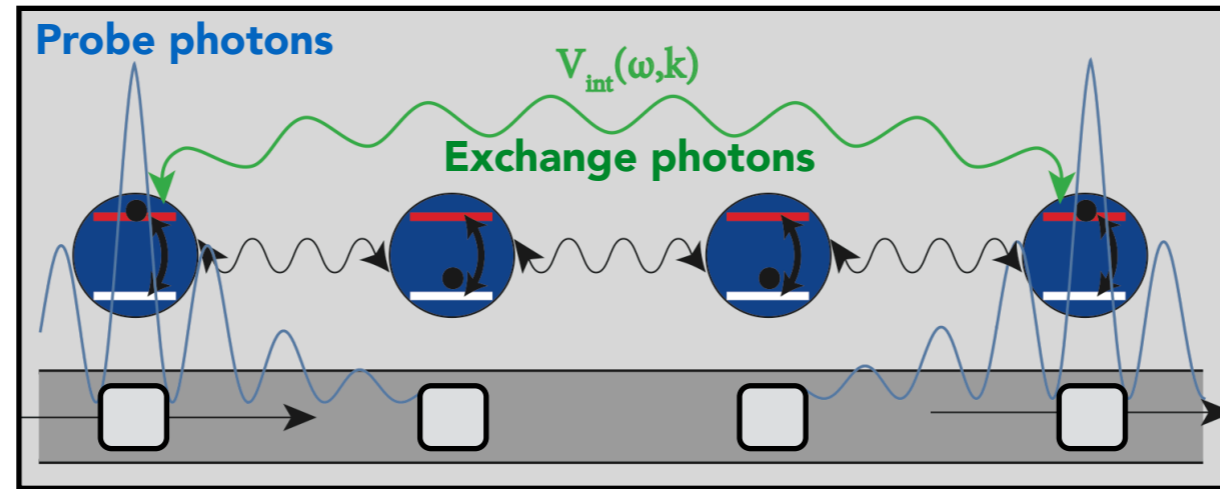
Dispersion:

$$\int \frac{dk}{2\pi} (\alpha_E (k - k_0)^2) \hat{a}_E^\dagger(k) \hat{a}_E(k)$$

Losses:

$$\kappa_E \mathcal{D}[\hat{a}_E(k)]$$

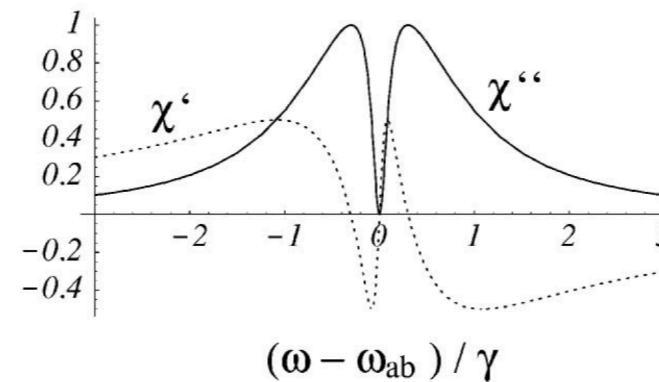
Probe photons propagation: Electromagnetically-Induced Transparency



Atomic levels

Fleischhauer, Lukin (2002)

$$\hat{H}_{\text{int}} = \sum_z \left[\Omega e^{-i\omega_L^{(1)}t} \hat{a}_e^\dagger(z) \hat{a}_s(z) + g_M \hat{a}_M(k) e^{ikz} u_k^M(z) \hat{a}_e^\dagger(z) \hat{a}_g(z) + h.c. \right]$$

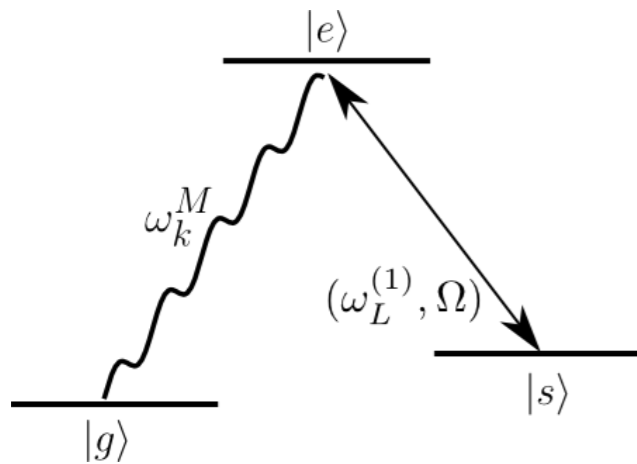


$$\omega_M(k) = \omega_{sg} - \omega_L^{(1)}$$

Large dispersion without absorption

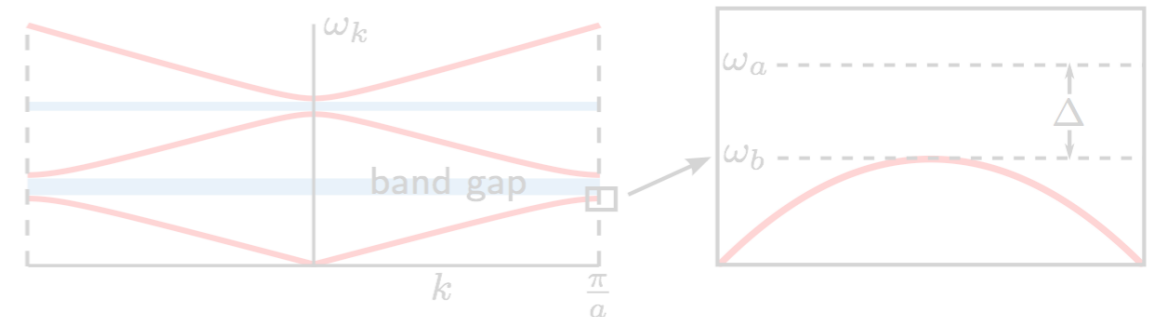
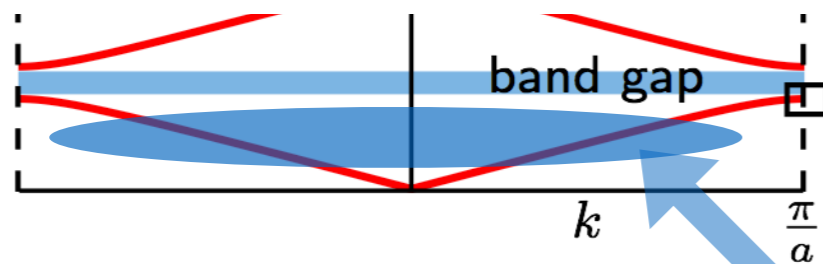
EIT propagation range:

$$L_M \simeq \frac{v_M}{\kappa_M}$$



Probe photons

Exchange photons



Losses:

$$\kappa_M \mathcal{D}[\hat{a}_M(k)]$$

Incoh. Drive:

$$\kappa_s \mathcal{P}[\hat{a}_M(k)]$$

Dispersion:

$$\int \frac{dk}{2\pi} (\alpha_E(k - k_0)^2) \hat{a}_E^\dagger(k) \hat{a}_E(k)$$

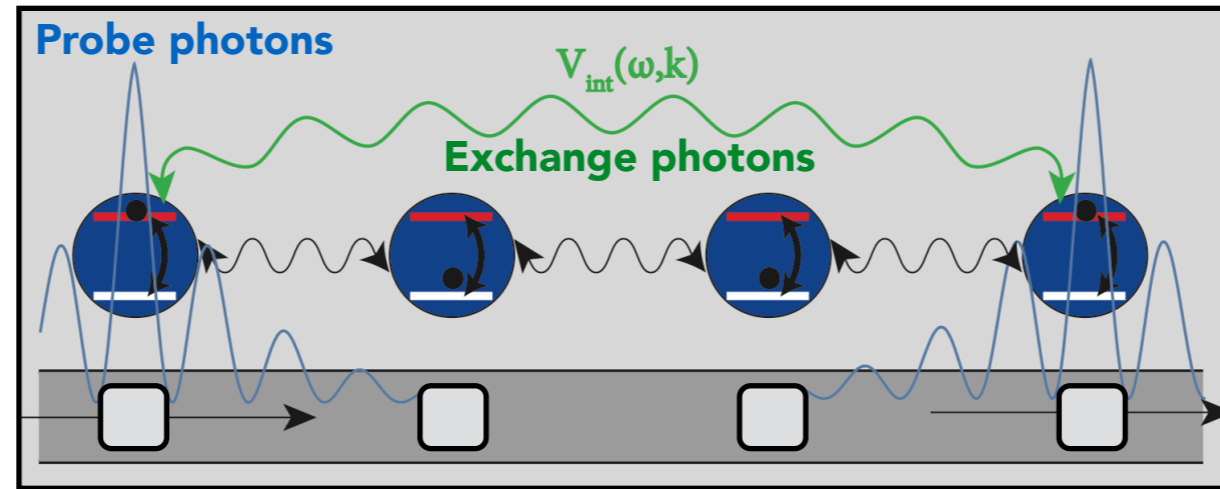
Losses:

$$\kappa_E \mathcal{D}[\hat{a}_E(k)]$$

Dispersion:

$$\int \frac{dk}{2\pi} J_M (1 - \cos(ka)) \hat{a}_M^\dagger(k) \hat{a}_M(k)$$

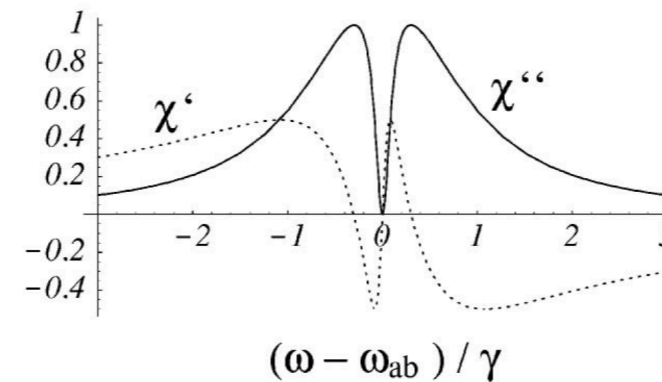
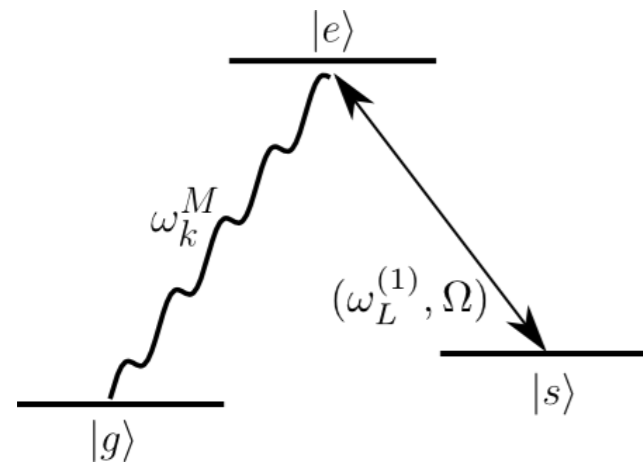
Probe photons propagation: Electromagnetically-Induced Transparency



Atomic levels

Fleischhauer, Lukin (2002)

$$\hat{H}_{\text{int}} = \sum_z \left[\Omega e^{-i\omega_L^{(1)}t} \hat{a}_e^\dagger(z) \hat{a}_s(z) + g_M \hat{a}_M(k) e^{ikz} u_k^M(z) \hat{a}_e^\dagger(z) \hat{a}_g(z) + h.c. \right]$$



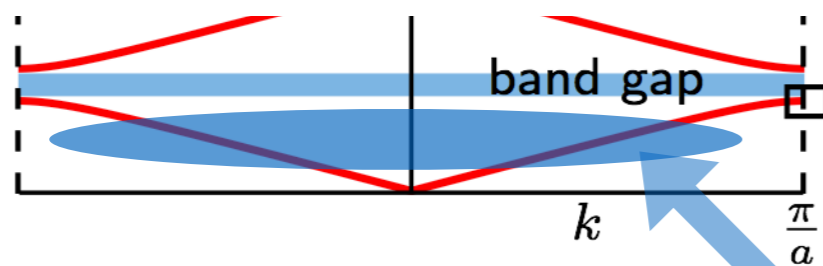
$$\omega_M(k) = \omega_{sg} - \omega_L^{(1)}$$

Large dispersion without absorption

EIT propagation range:

$$L_M \simeq \frac{v_M}{\kappa_M}$$

Probe photons



Losses:

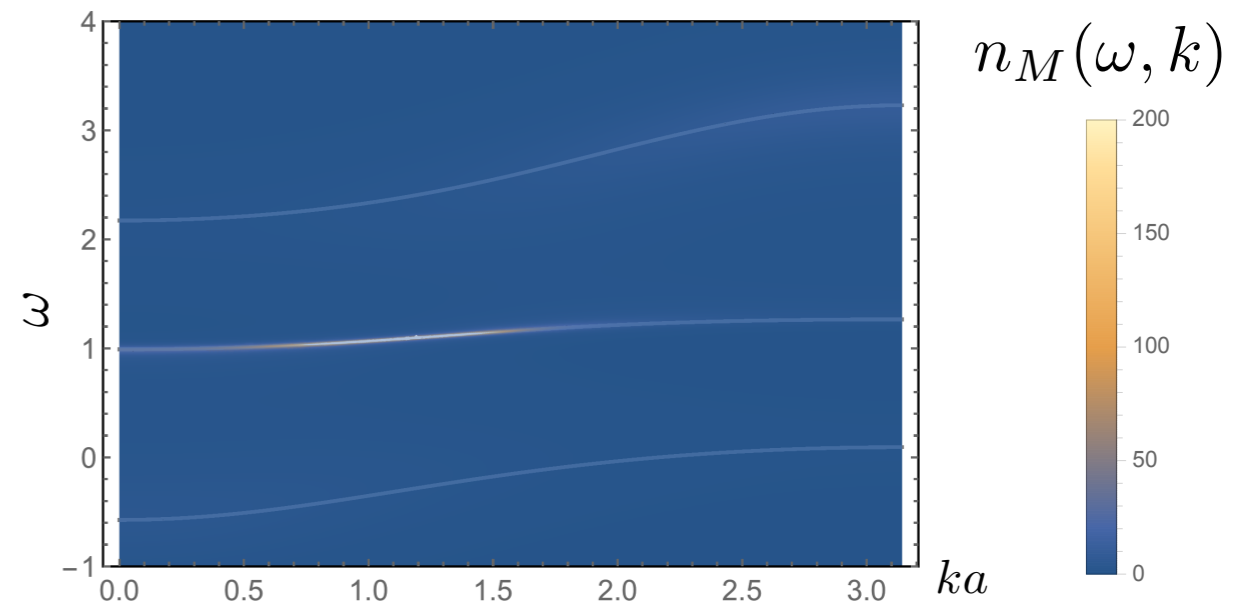
$$\kappa_M \mathcal{D}[\hat{a}_M(k)]$$

Dispersion:

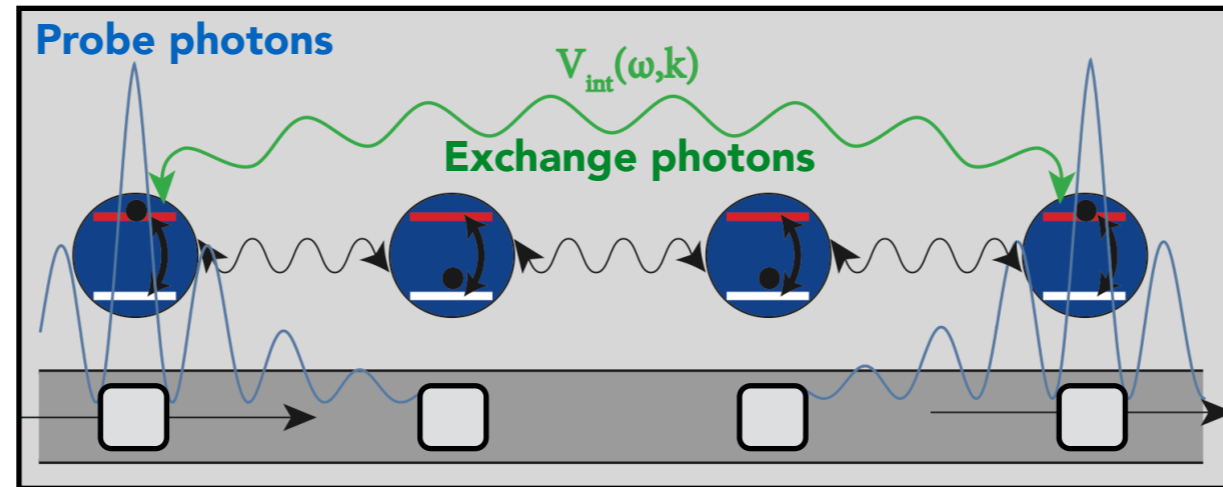
$$\int \frac{dk}{2\pi} J_M (1 - \cos(ka)) \hat{a}_M^\dagger(k) \hat{a}_M(k)$$

Incoh. Drive:

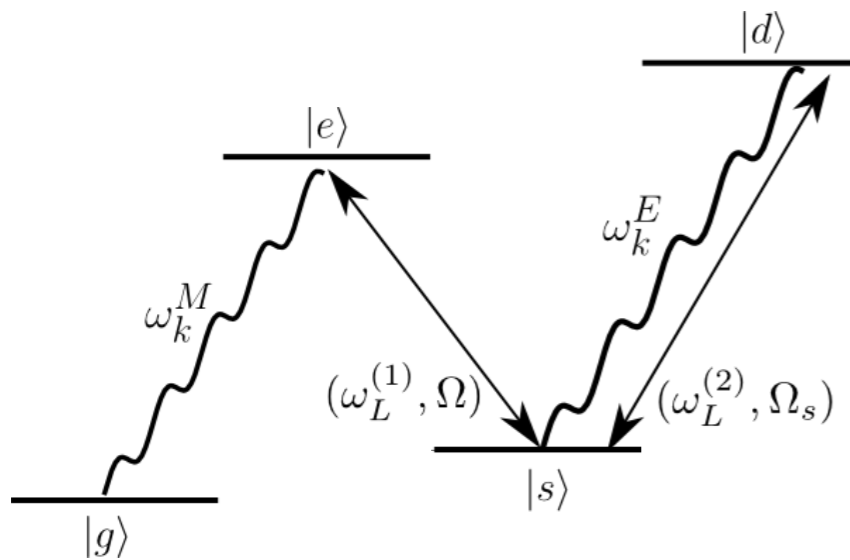
$$\kappa_s \mathcal{P}[\hat{a}_M(k)]$$



Interacting EIT polaritons



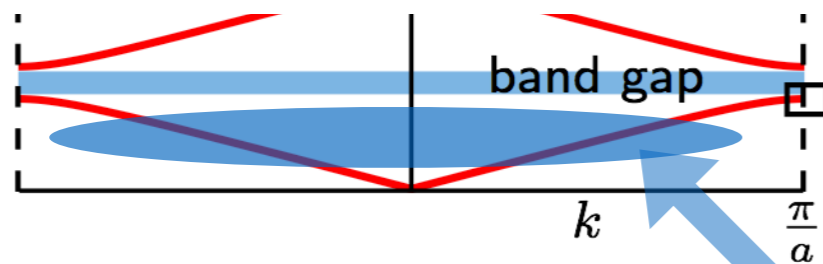
Atomic levels



EIT propagation range: $L_M \simeq \frac{v_M}{\kappa_M}$

Exponential interaction range: $L_E \simeq \sqrt{\frac{\alpha_E}{\Delta}}$

Probe photons



Dispersion:

$$\int \frac{dk}{2\pi} J_M (1 - \cos(ka)) \hat{a}_M^\dagger(k) \hat{a}_M(k)$$

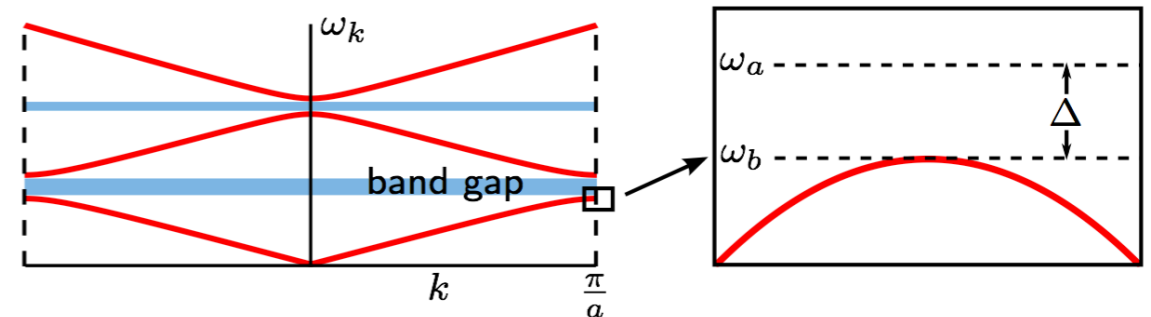
Losses:

$$\kappa_M \mathcal{D}[\hat{a}_M(k)]$$

Incoh. Drive:

$$\kappa_s \mathcal{P}[\hat{a}_M(k)]$$

Exchange photons



Dispersion:

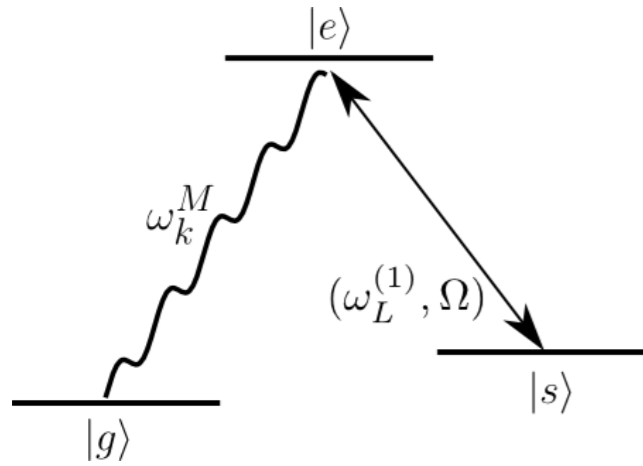
$$\int \frac{dk}{2\pi} (\alpha_E (k - k_0)^2) \hat{a}_E^\dagger(k) \hat{a}_E(k)$$

Losses:

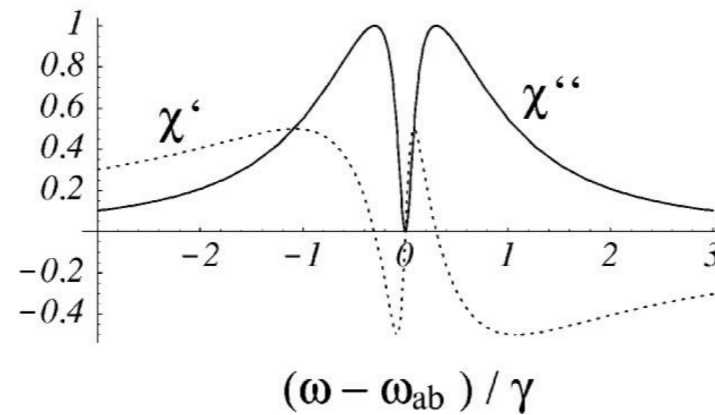
$$\kappa_E \mathcal{D}[\hat{a}_E(k)]$$

Non-equilibrium diagrammatic approach to EIT

Atomic levels



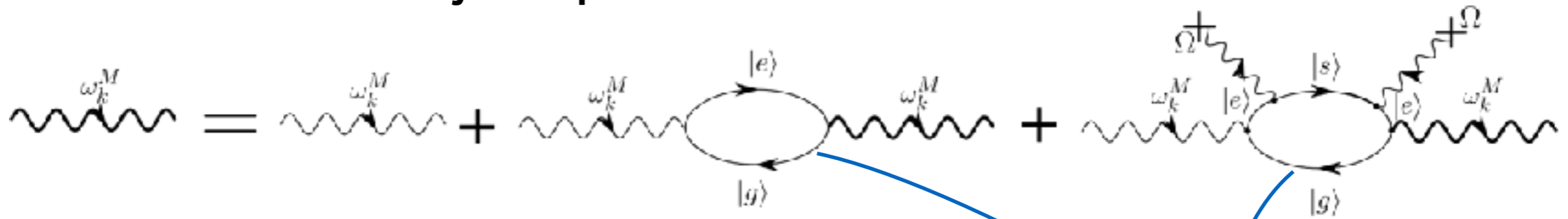
$$\hat{H}_{\text{int}} = \sum_z \left[\Omega e^{-i\omega_L^{(1)}t} \hat{a}_e^\dagger(z) \hat{a}_s(z) + g_M \hat{a}_M(k) e^{ikz} u_k^M(z) \hat{a}_e^\dagger(z) \hat{a}_g(z) + h.c. \right]$$



Large dispersion without absorption

EIT propagation range:

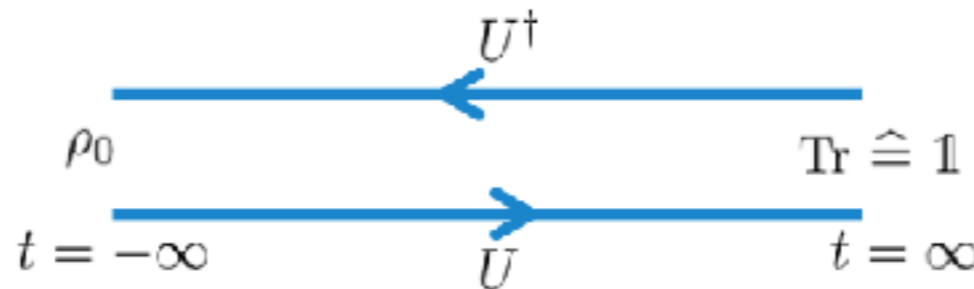
Dyson equations for Green's function:



$$\chi(\omega, k) = \frac{g_M^2 |u_k^M(0)|^2}{\omega - \omega_e - \frac{\Omega^2}{\omega - \omega_s - \omega_L^{(1)} + i\gamma_e/2}}$$

destructive interference = EIT

Keldysh real-time contour:

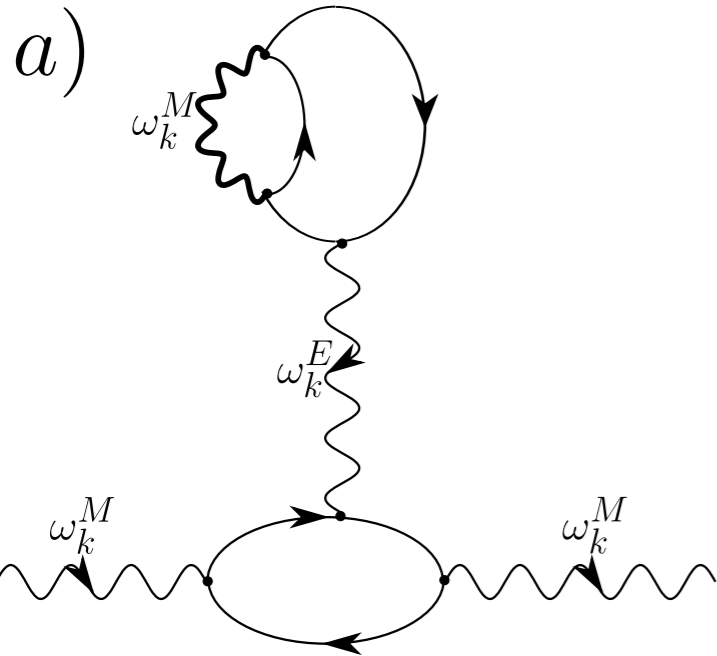


Forward-backward branch:

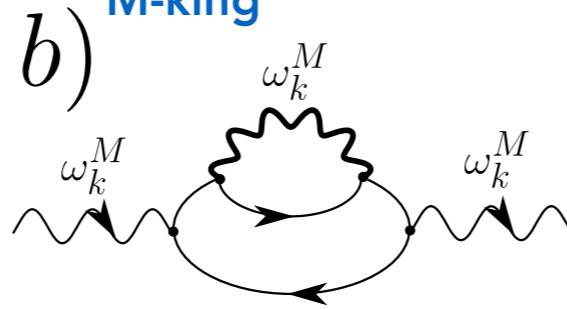
double # of GF's

Controlled 1/range expansion for the interaction diagrams

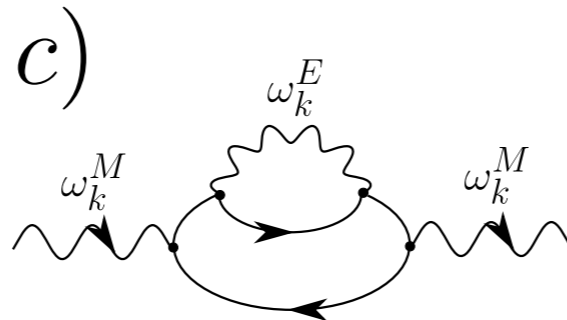
"Alien"



"M-king"



"E-king"

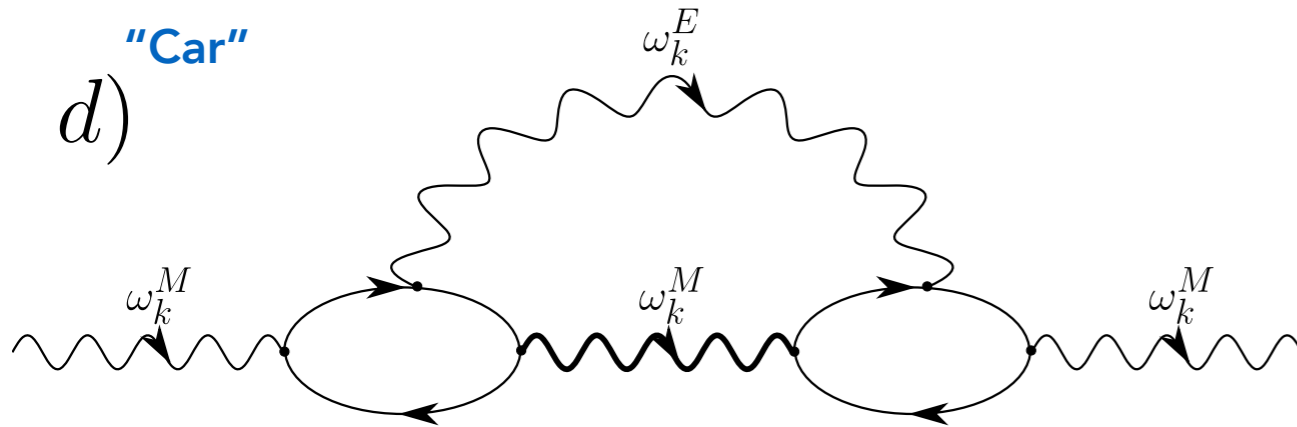


- Expansion for large propagation range

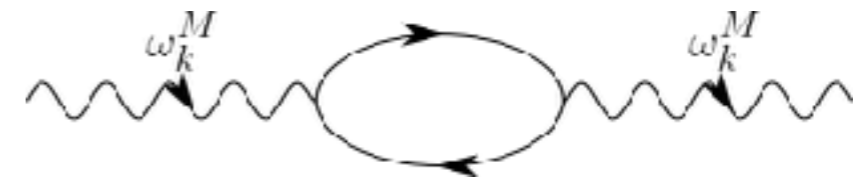
Effective coupling scales like $1/\sqrt{L}$:
multiple scattering suppressed

$$\left(\frac{a}{L}\right)^{\#\text{loops} - \#\text{atom-loops}}$$

"Car"



- EIT is leading order $O(1)$:

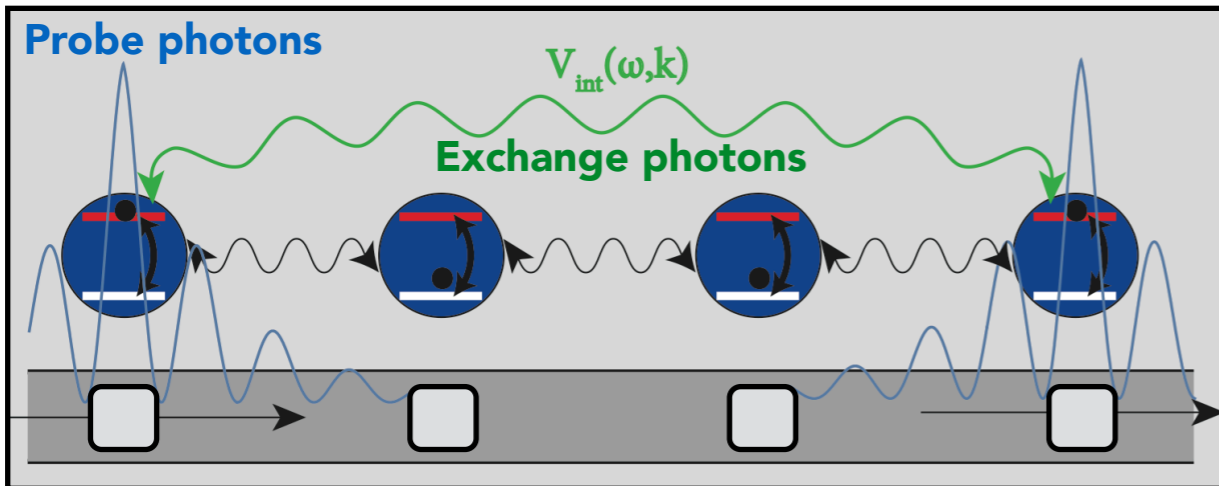


All next-to-leading order $O(1/L)$ diagrams

for probe photons

(Keldysh structure and atom internal structure not shown)

Parameter regimes for selected interaction processes



Tuneability:

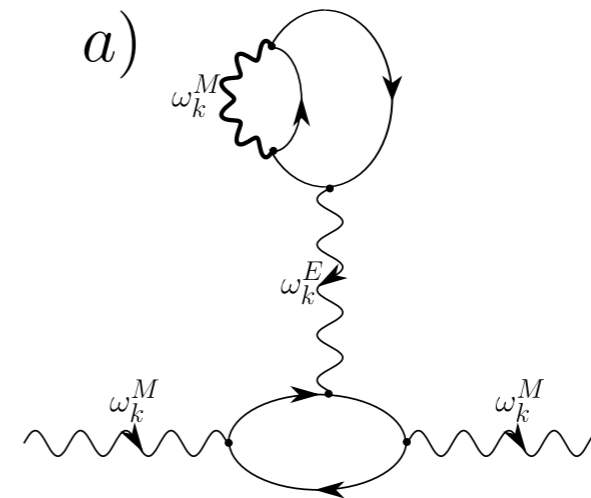
EIT propagation range: $L_M \simeq \frac{v_M}{\kappa_M}$

Exponential interaction range: $L_E \simeq \sqrt{\frac{\alpha_E}{\Delta}}$

Still very complex theory at order a/L :

- Three-loop (leading interaction contribution)
- 4 atom + 2 photon degrees of freedom
- Keldysh structure

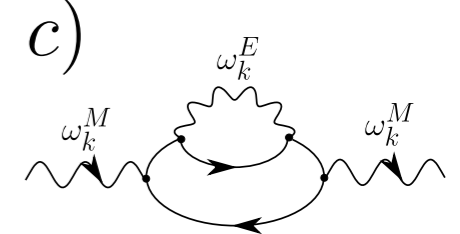
"Alien"



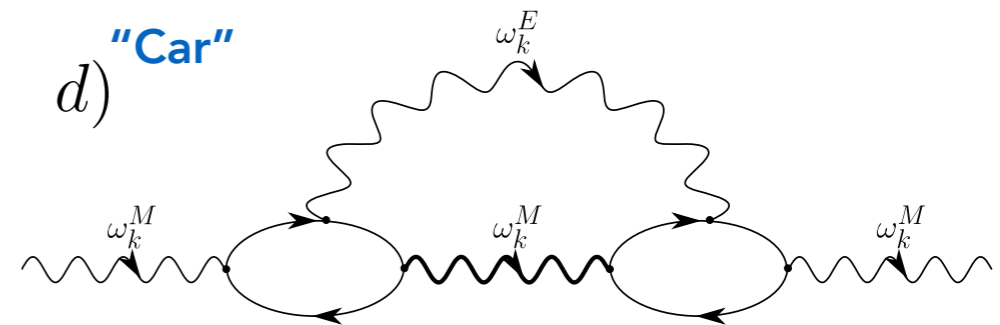
"M-king"



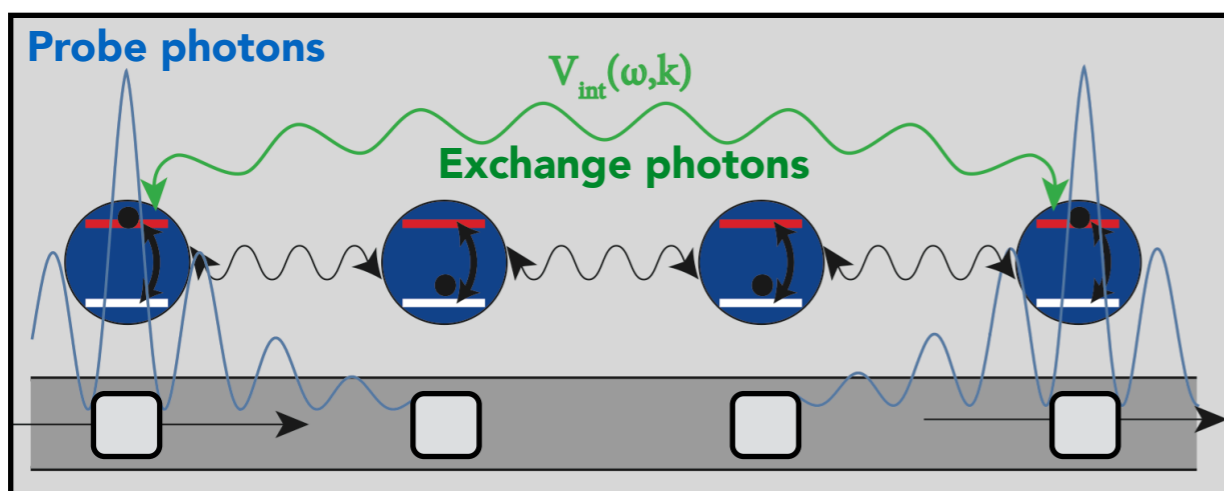
"E-king"



"Car"



Parameter regimes for selected interaction processes



Tuneability:

EIT propagation range: $L_M \simeq \frac{v_M}{\kappa_M}$

Exponential interaction range: $L_E \simeq \sqrt{\frac{\alpha_E}{\Delta}}$

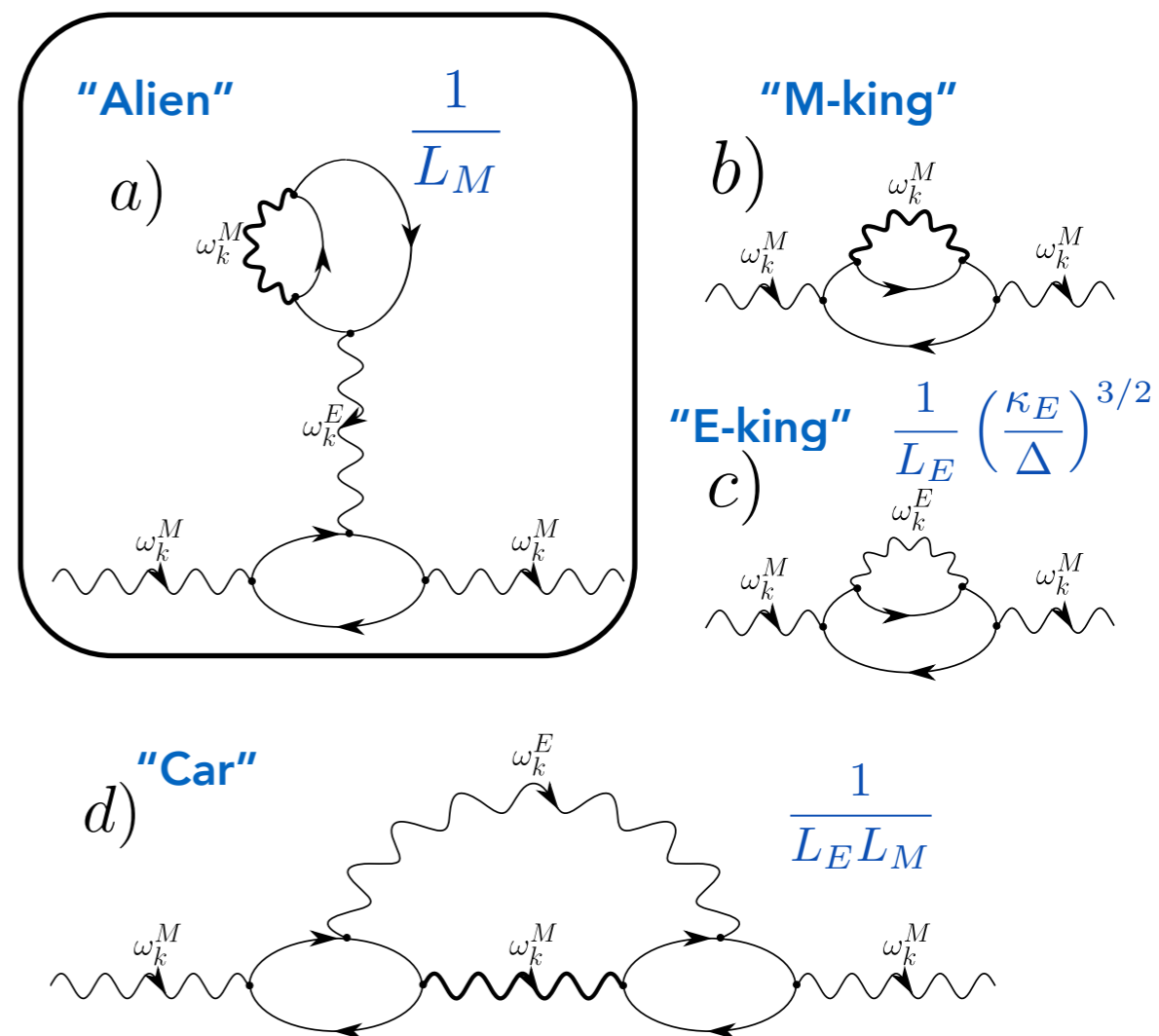
Large interaction range: $\frac{L_E}{L_M} \gg \left| \frac{\kappa_E}{\Delta} \right|^{3/2}$

Interaction-induced shifts dominate over scattering

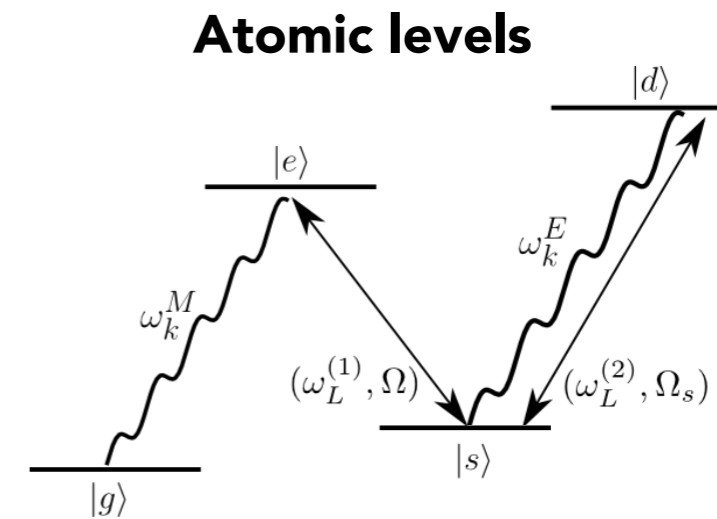
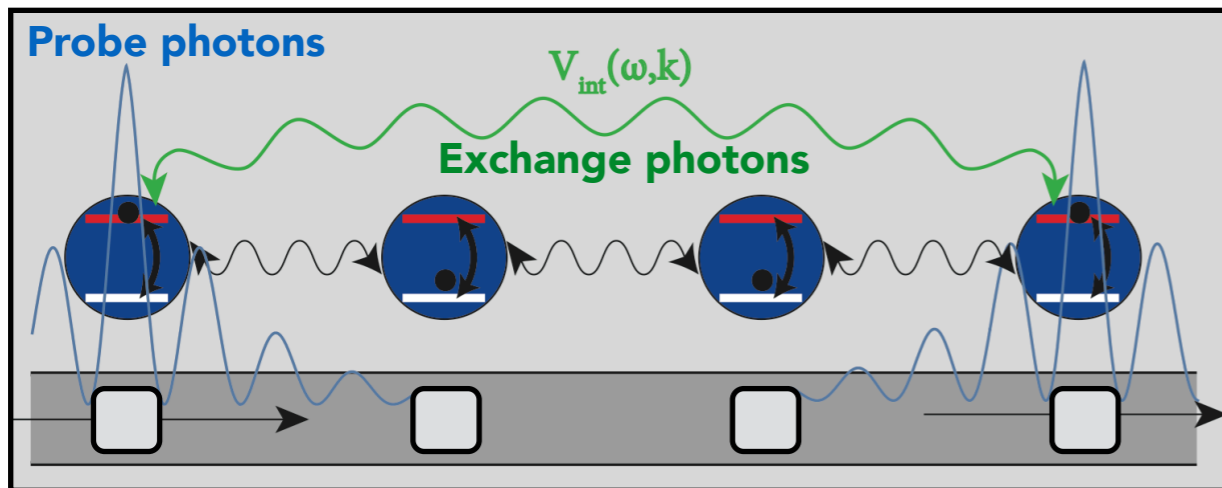
"Alien" (Hartree) diagram dominates

Still very complex theory at $1/L$:

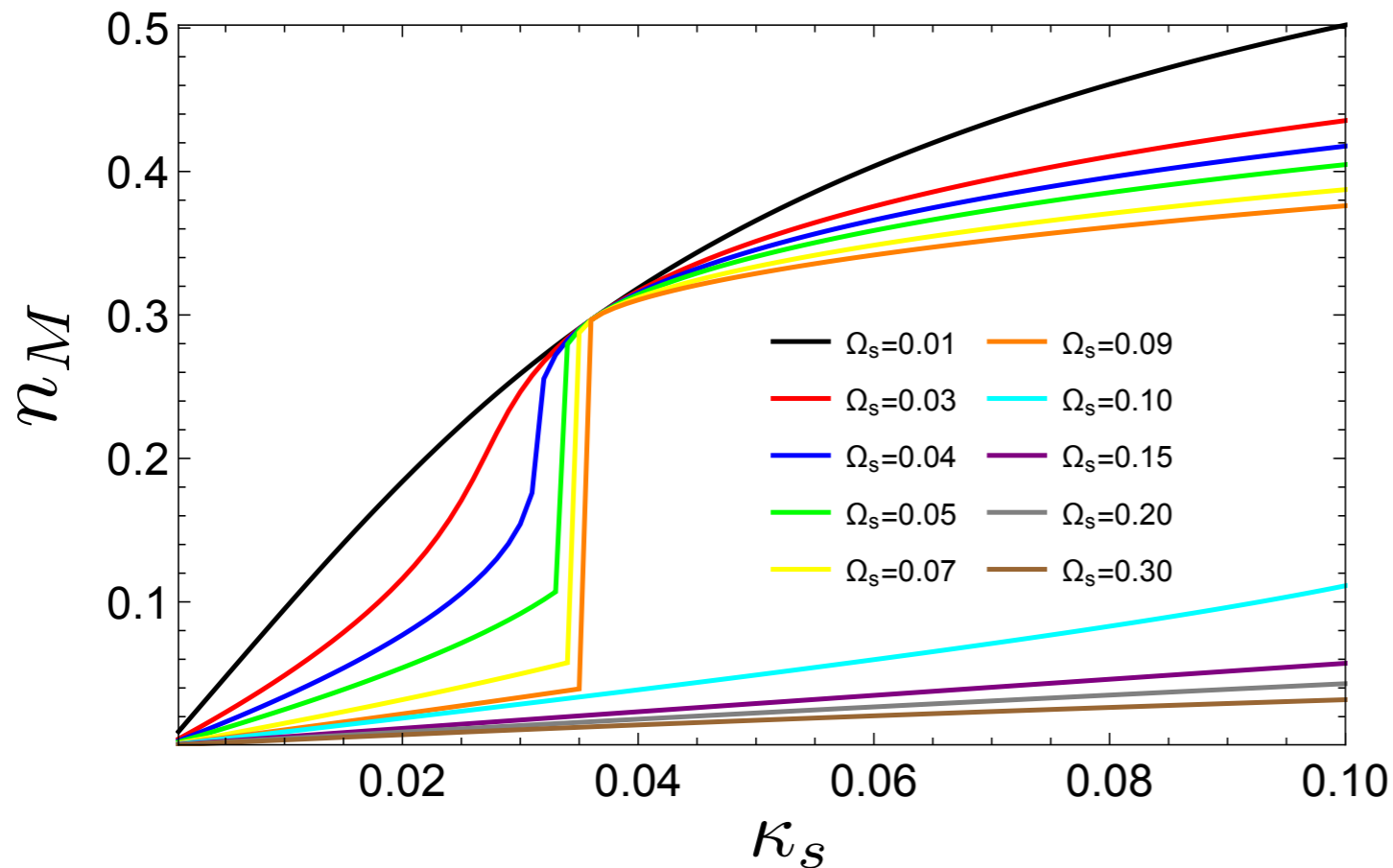
- Three-loop (leading interaction contribution)
- 4 atom + 2 photon degrees of freedom
- Keldysh structure



Non-equilibrium phase transition



Number of probe photons in the steady state

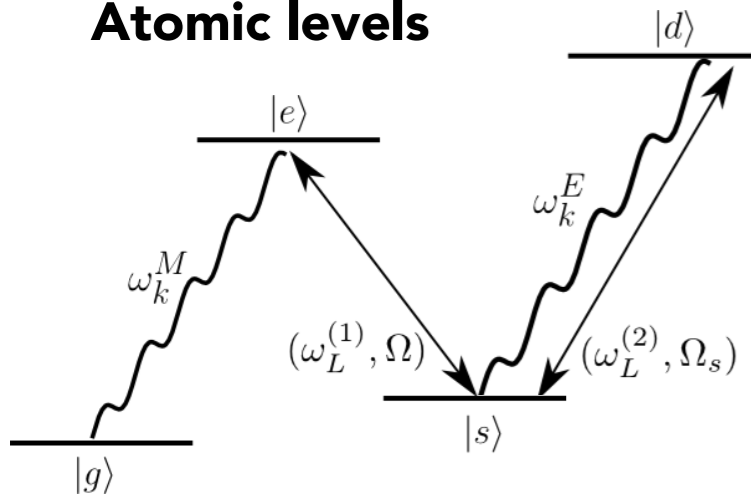


- **Transition between bright and dark phase**
First order, culminates in bi-critical point

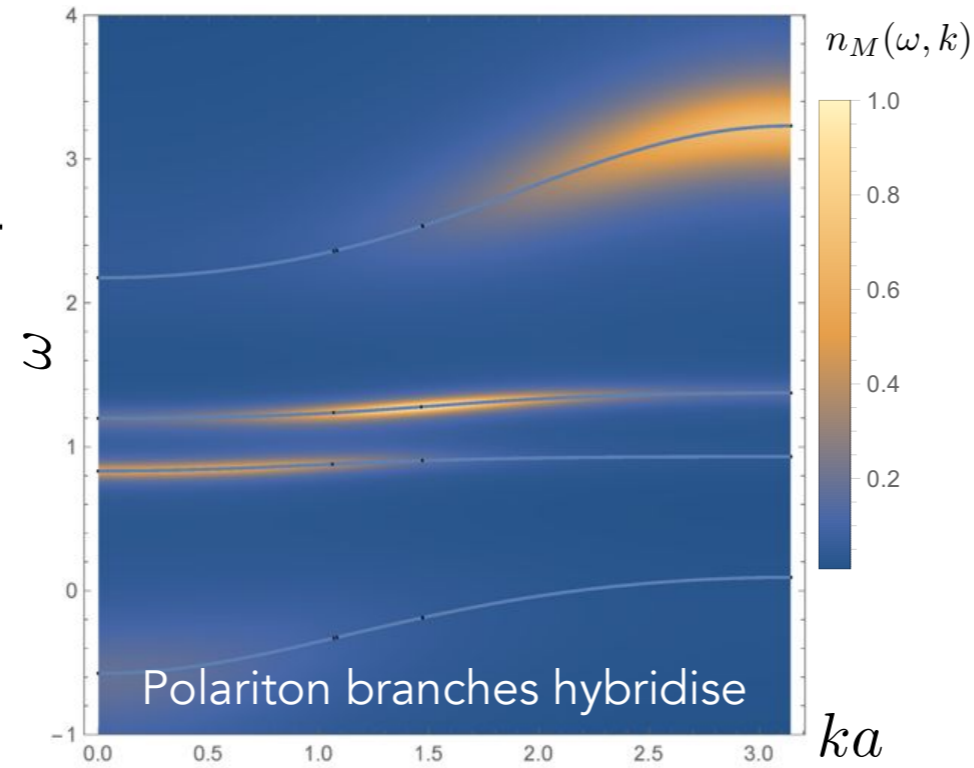
Interaction-induced transparency

Probe-photons occupation:

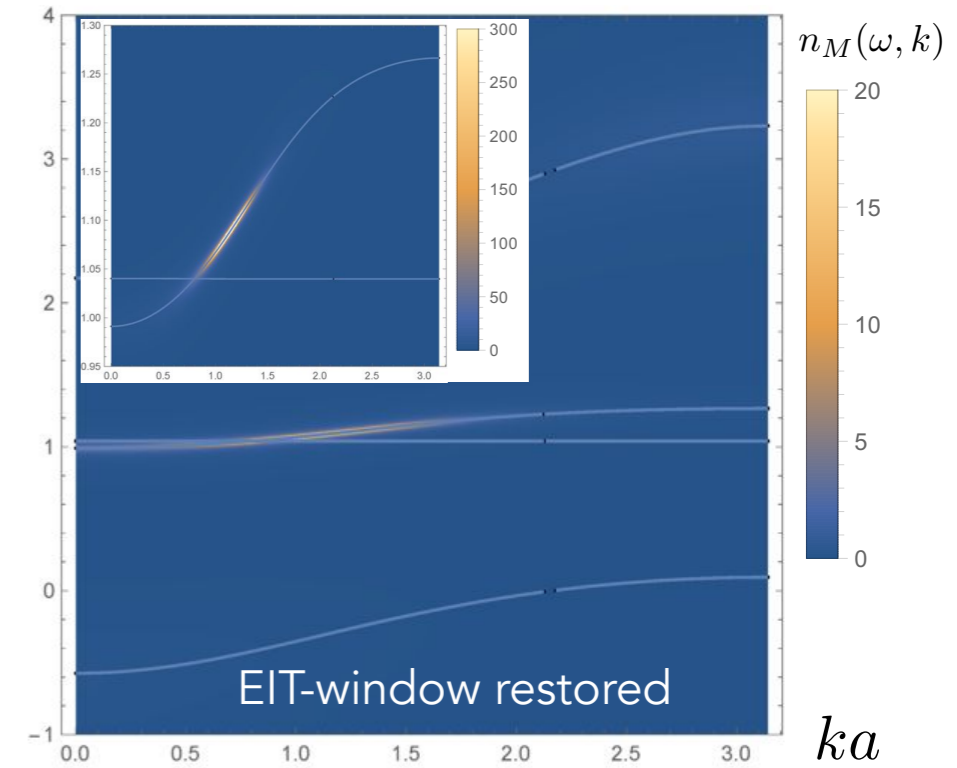
Atomic levels



Dark phase



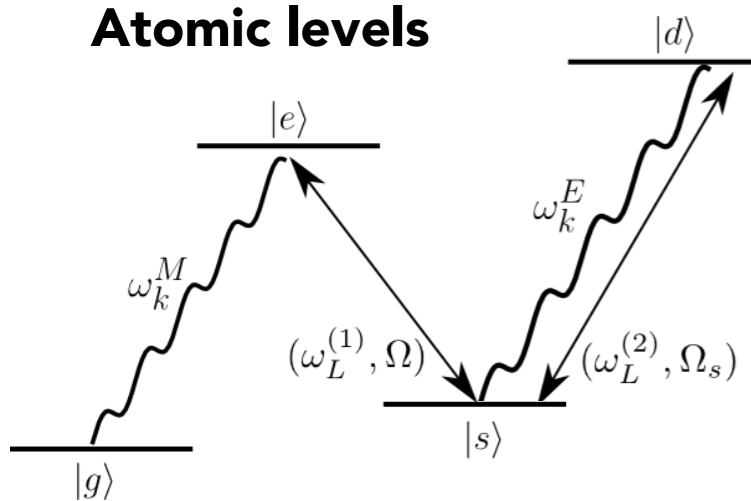
Bright phase



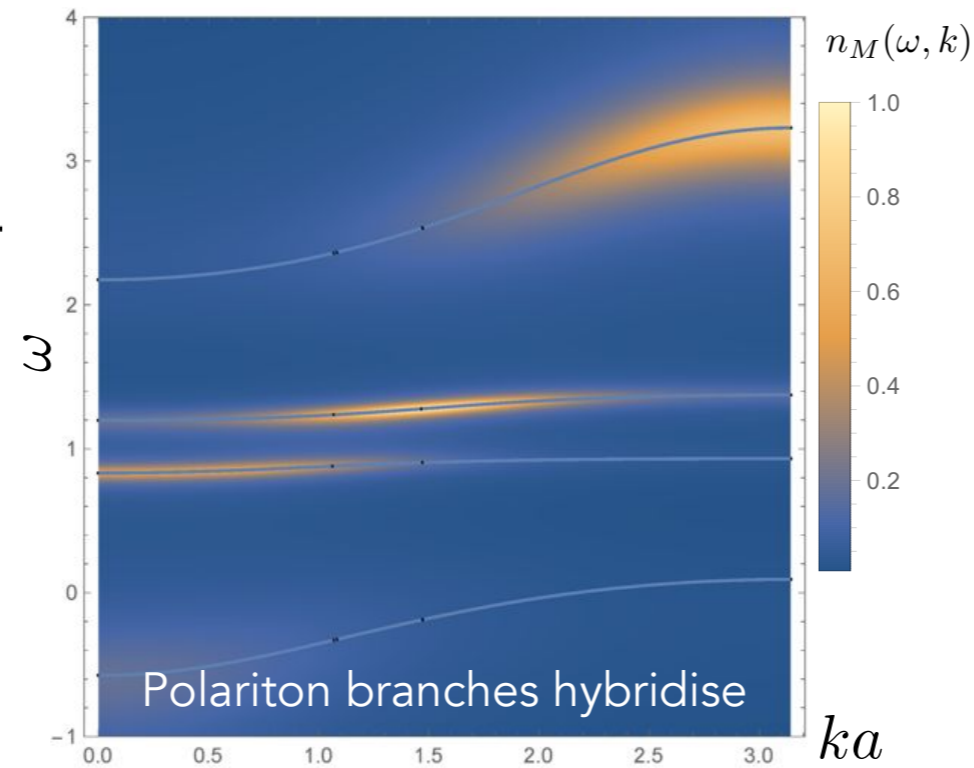
Interaction-induced transparency

Probe-photons occupation:

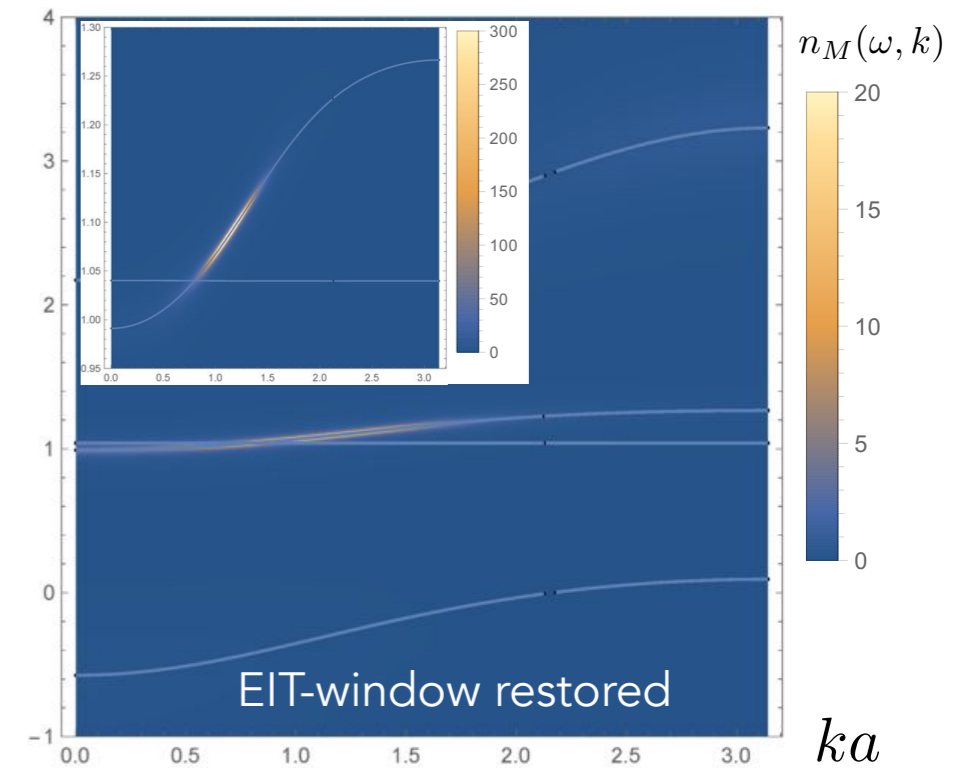
Atomic levels



Dark phase



Bright phase



Electromagnetically Induced Transparency (EIT):

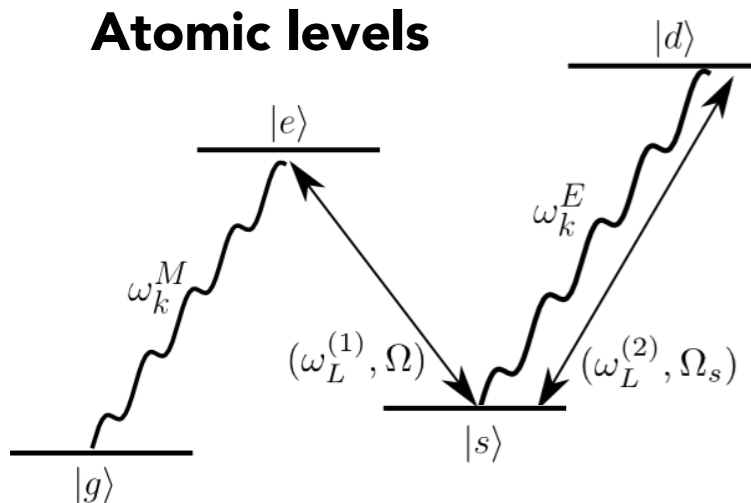
$$\chi_{\text{EIT}} = \text{Diagram 1} + \text{Diagram 2}$$

Diagrammatic representation of the EIT susceptibility. The first term is a loop between states $|g\rangle$ and $|e\rangle$. The second term is a loop between states $|g\rangle$ and $|s\rangle$ with an additional transition to $|e\rangle$ and back, involving Rabi frequency Ω .

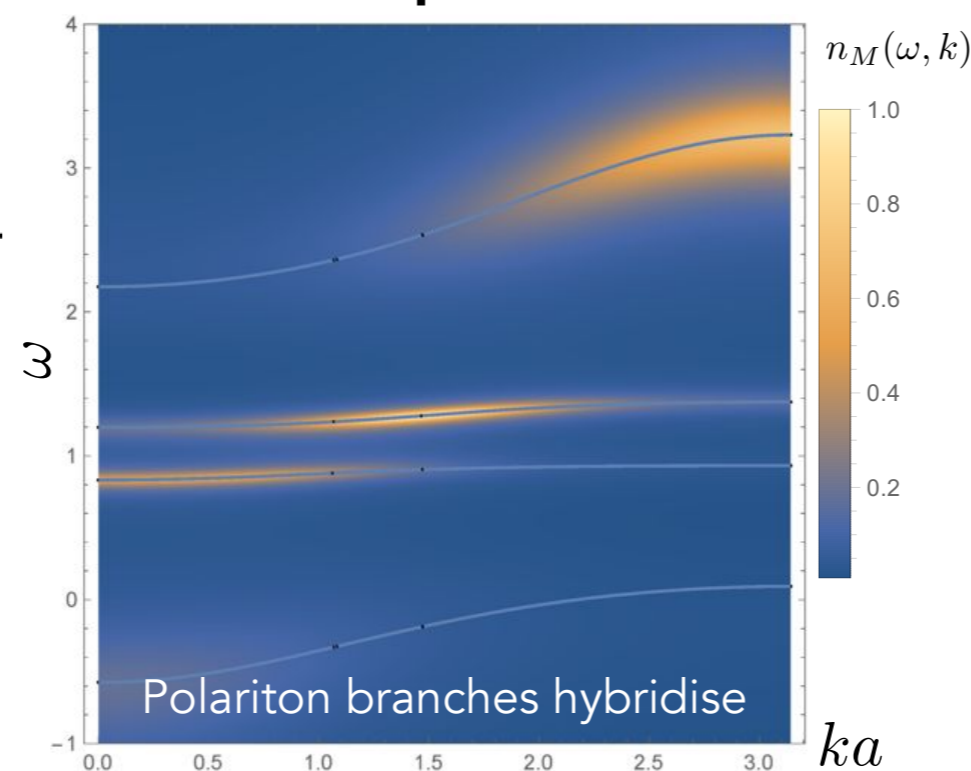
Interaction-induced transparency

Probe-photons occupation:

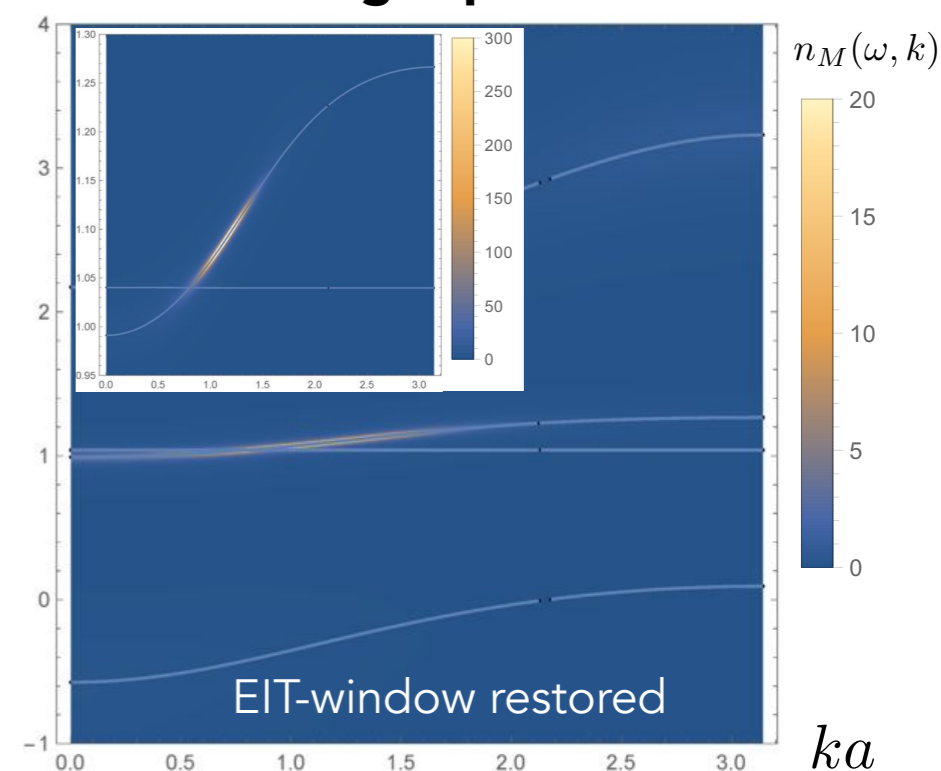
Atomic levels



Dark phase



Bright phase



Electromagnetically Induced Transparency (EIT):

$$\chi_{\text{EIT}} = \text{Diagram 1} + \text{Diagram 2}$$

Diagram 1: Loop with states $|g\rangle$ and $|e\rangle$.
Diagram 2: Loop with states $|g\rangle$ and $|s\rangle$, with external fields Ω and Ω_s and a transition to $|e\rangle$.

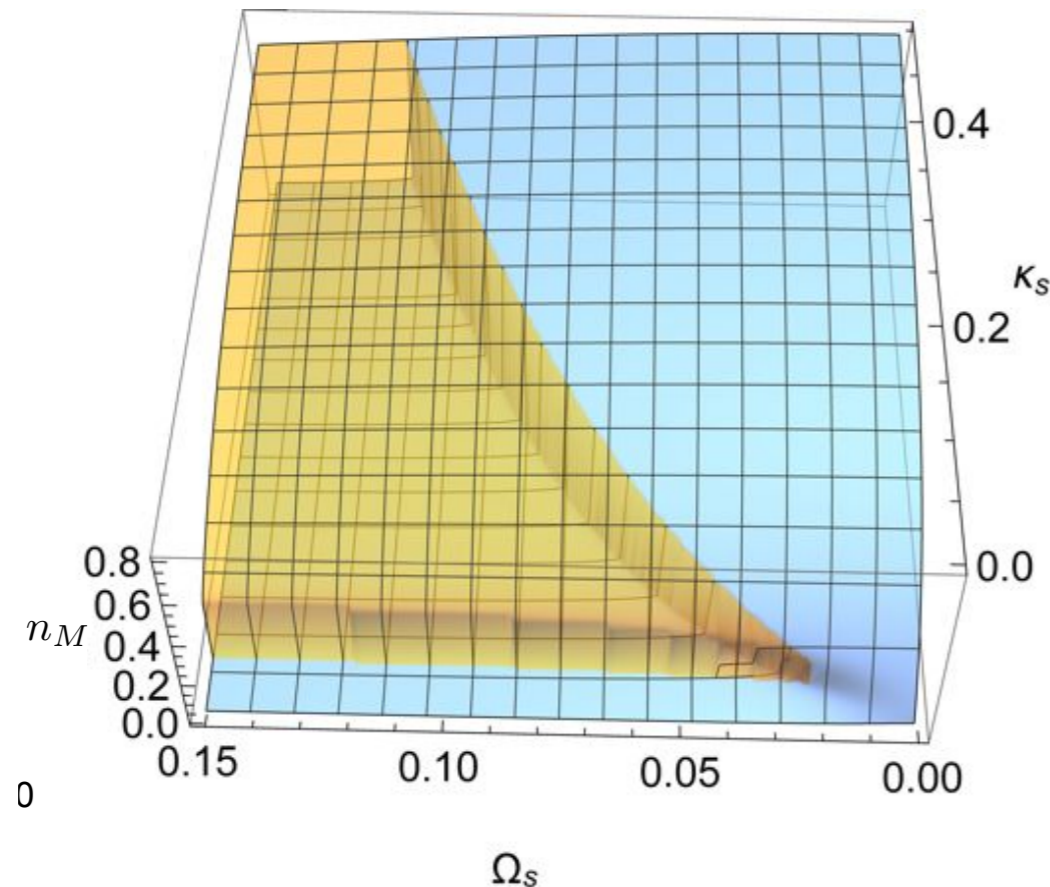
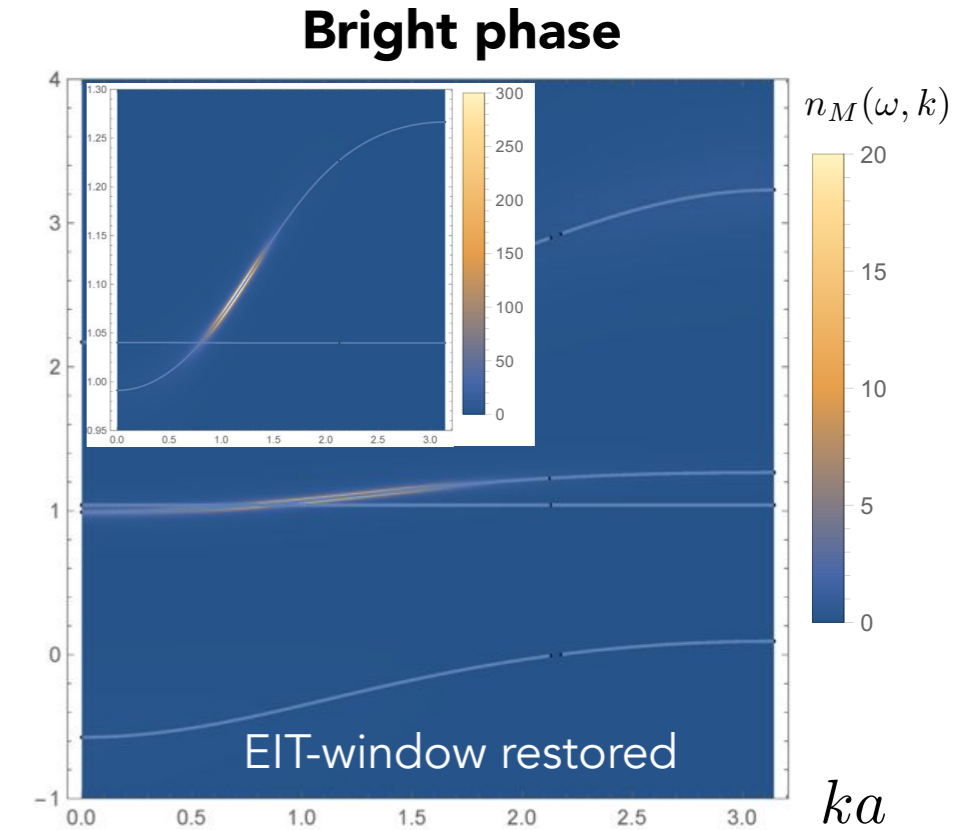
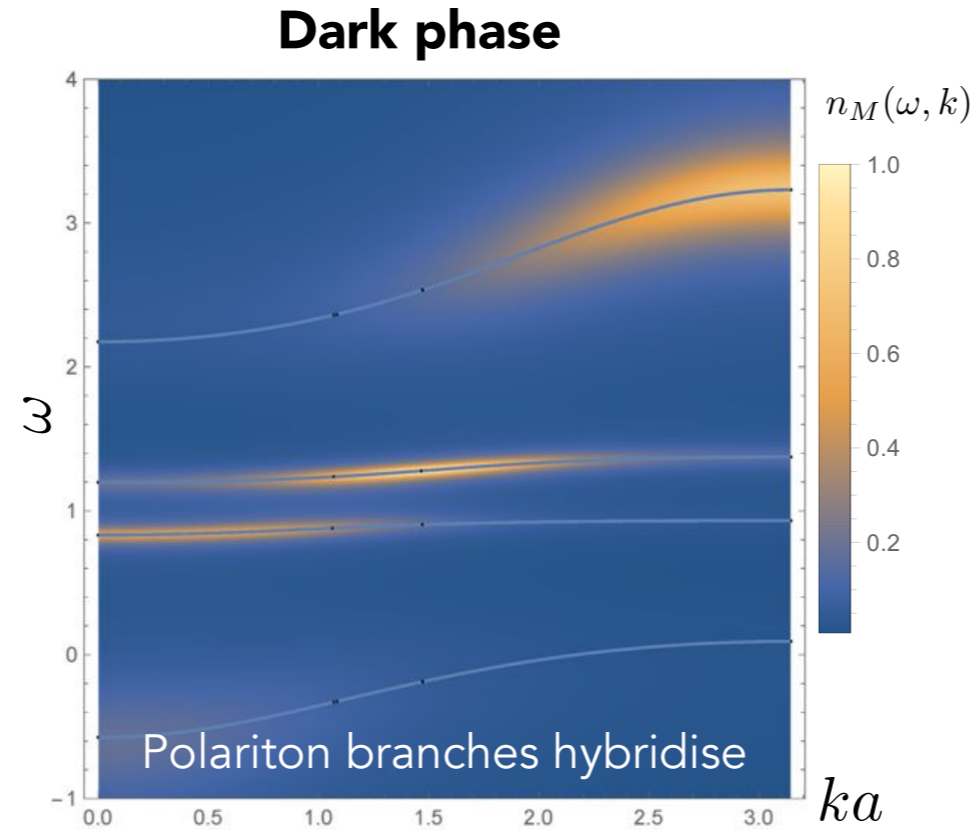
Interaction-Induced Transparency (IIT):

$$\chi_{\text{IIT}} = \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6}$$

Diagram 3: Loop with states $|s\rangle$ and $|d\rangle$, with external fields Ω_s and Ω_s .
Diagram 4: Loop with states $|s\rangle$ and $|d\rangle$, with external fields Ω_s and Ω_s , and a transition to $|e\rangle$.
Diagram 5: Loop with states $|s\rangle$ and $|d\rangle$, with external fields Ω_s and Ω_s , and a transition to $|e\rangle$.
Diagram 6: Loop with states $|s\rangle$ and $|d\rangle$, with external fields Ω_s and Ω_s , and a transition to $|e\rangle$.

Bistability

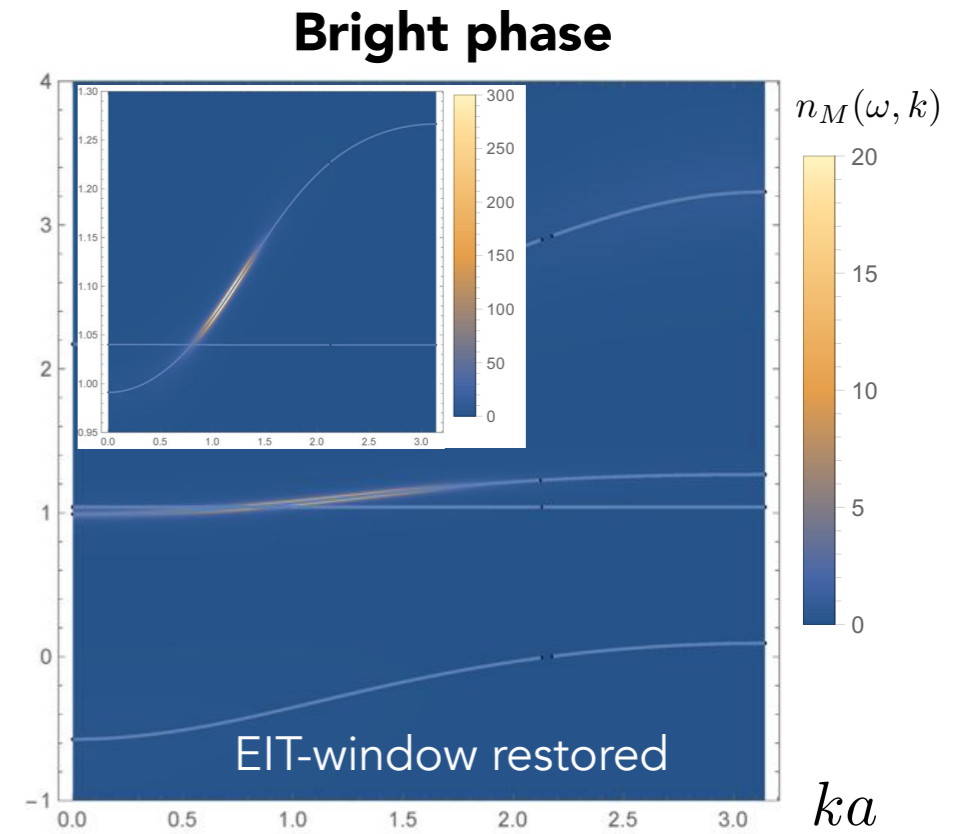
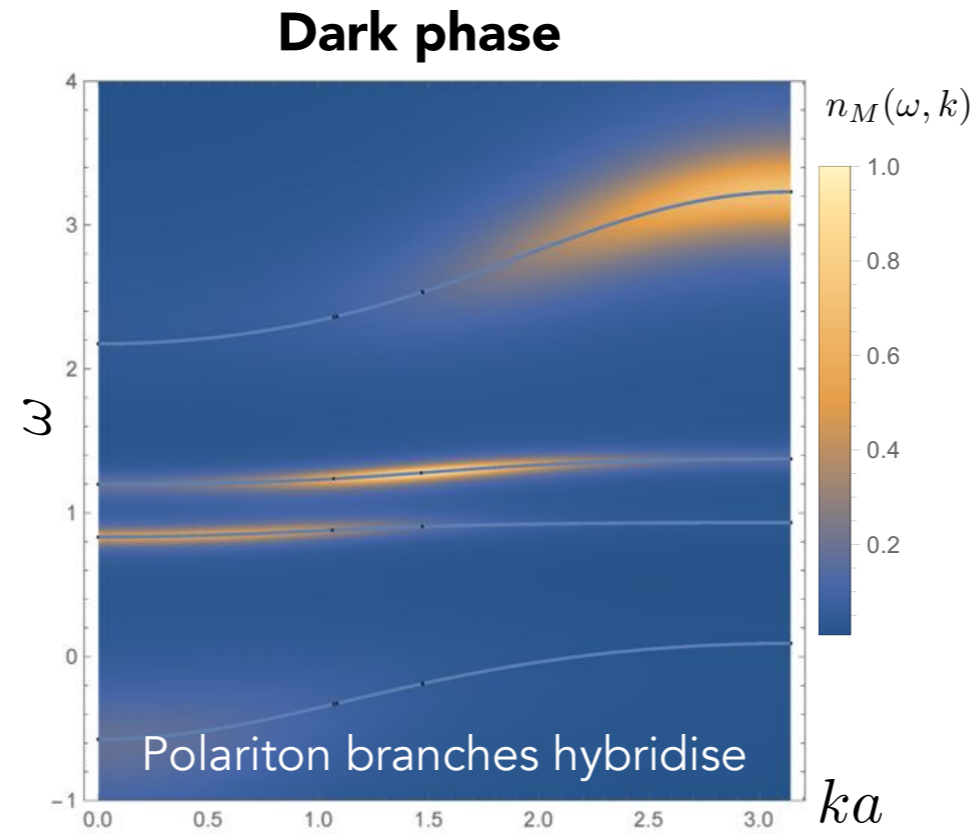
Probe-photons occupation:



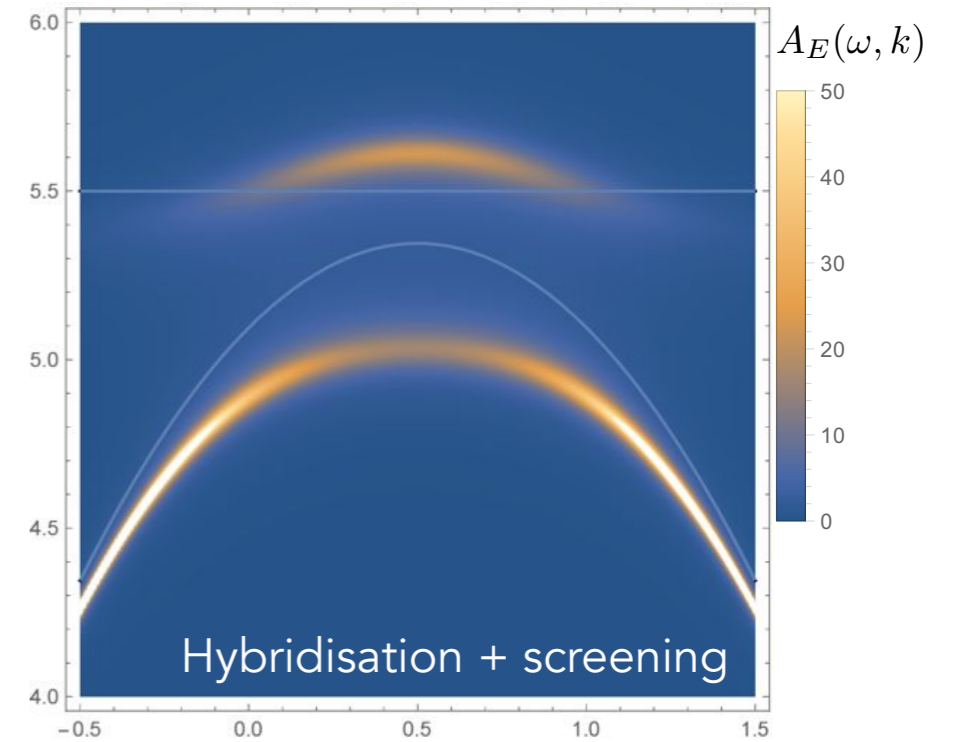
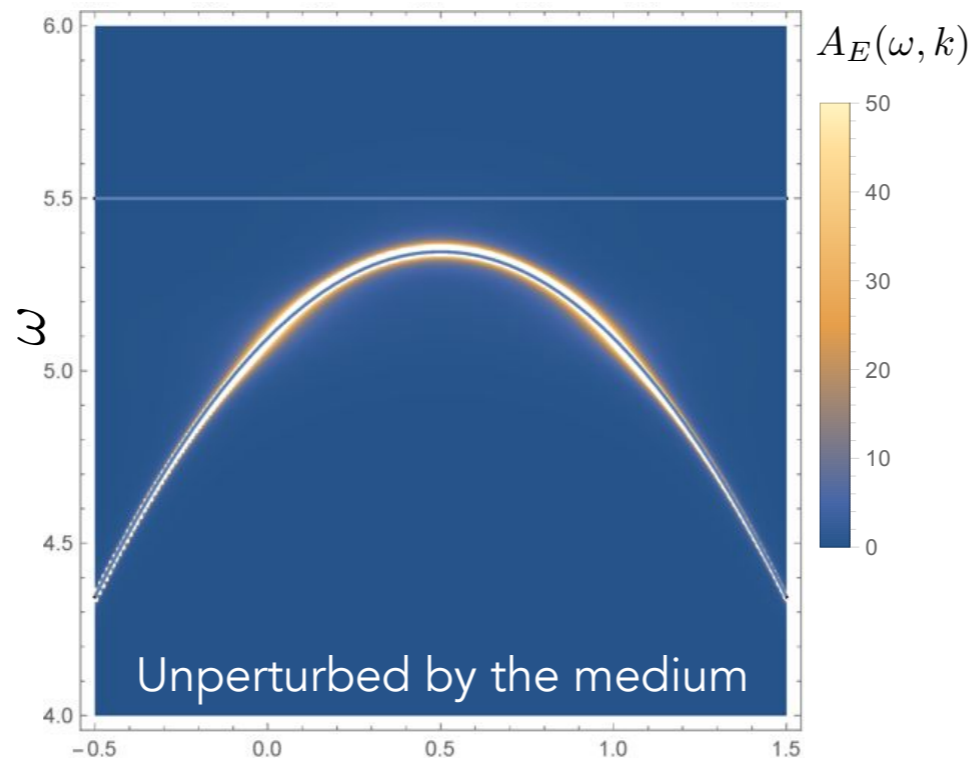
- **Transition between bright and dark phase**
First order, culminates in bi-critical point
- **Interactions can restore the EIT window**
Destructive interference between s-d excitation paths involving exchange photons
- **Bistability between dark and bright phase**
One interference path involves polariton interactions i.e. is nonlinear

Screening effects

Probe-photons occupation:



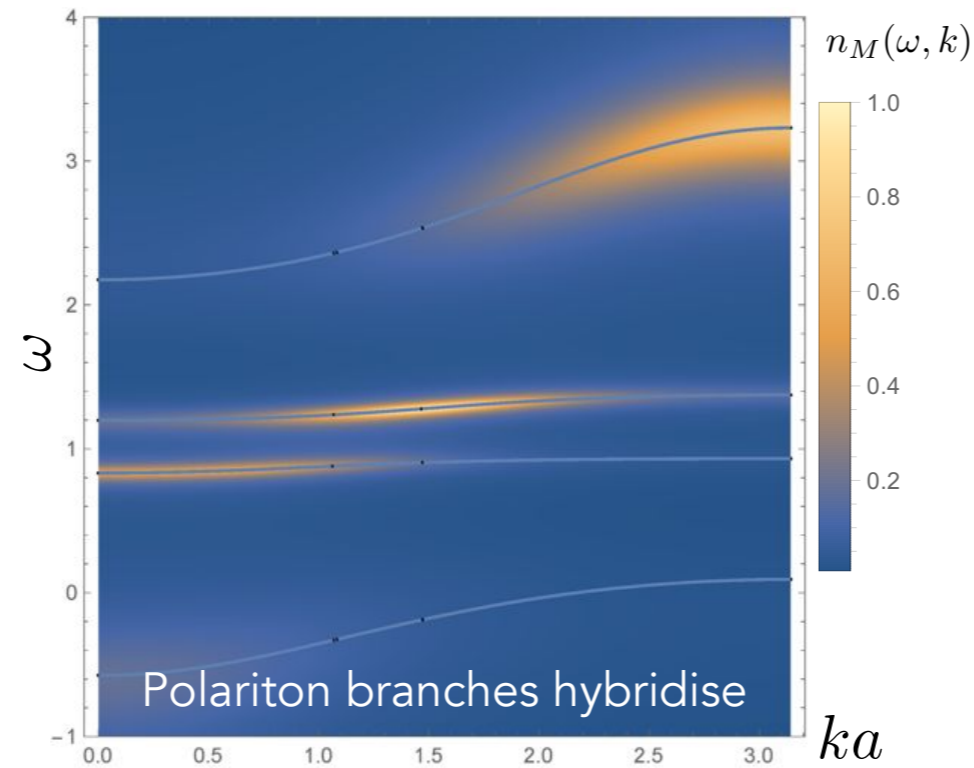
Exchange-photons spectrum:



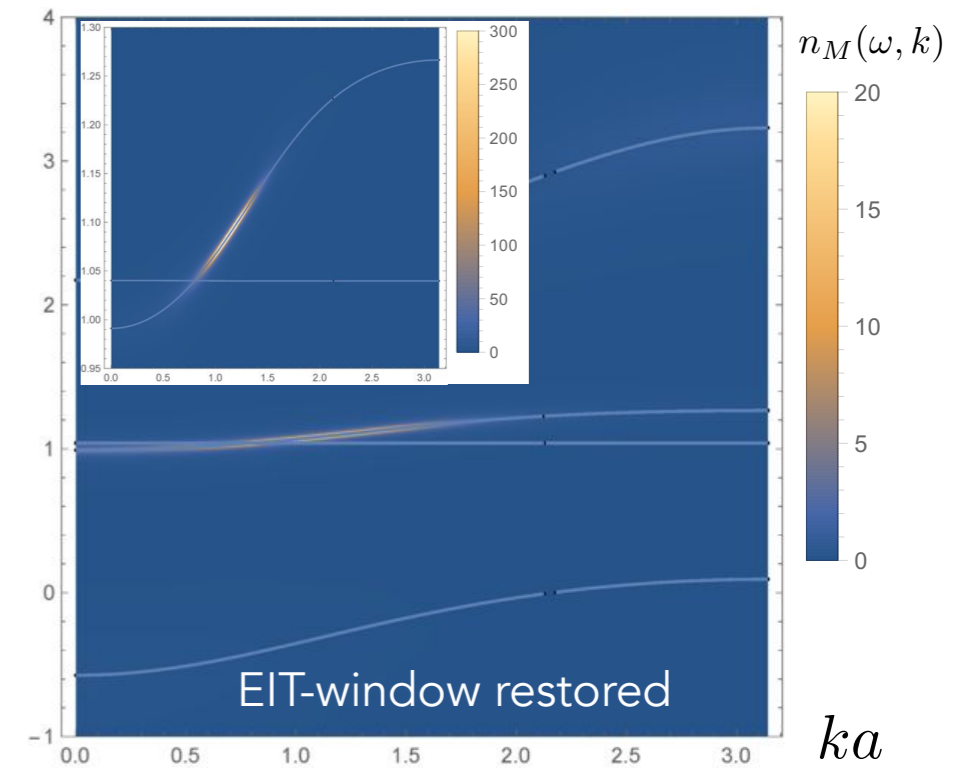
Intrinsic non-equilibrium nature

Probe-photons occupation:

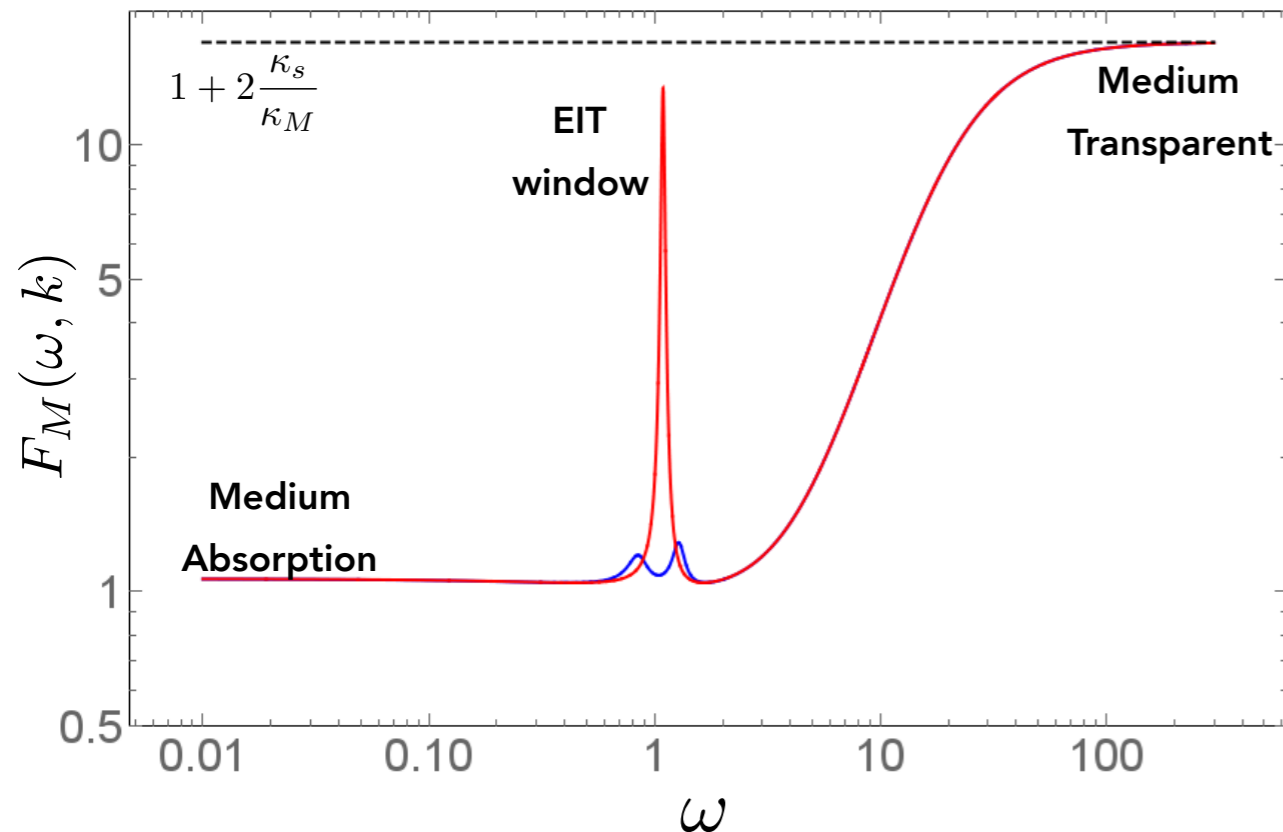
Dark phase



Bright phase



Probe-photons distribution function:



No effective thermal equilibrium

- EIT window emerges between Markov regions of total transparency and total absorption
- EIT peak is a sharp Lorentzian: no effective thermalisation

Summary

1. Controlled diagrammatic approach to strongly interacting EIT polaritons

- A. Non-perturbative diagrammatic expansion in $1/\text{range}$ for non-equilibrium GFs
- B. Tuneability of photon dispersion allows to select interaction processes
- C. Identify parameter regimes for quantitative relevance

2. Non-equilibrium phase transition on the EIT-window

- D. Transition between a dark and bright phase where interactions restore the EIT-window
- E. Bistability culminating in bi-critical point
- F. The EIT effect makes the transition of non-equilibrium nature

Outlook

- i) Dynamics of inhomogeneously driven system: How to ideally enter the bright phase experimentally?
- ii) Universality class of the non-equilibrium transition
- iii) Include scattering diagrams and treat photon crystallization.