СОФИЙСКИ УНИВЕРСИТЕТ „СВ. КЛИМЕНТ ОХРИДСКИ" ОСНОВАН 1888 Г.

Eigenstate thermalization in Floquet dynamics

Teodora Serafimova

Research focus

Isolated MB interacting quantum system,
driven by sudden periodic quenches

- How does one real Floquet system behave in the intuitive limit of short driving periods, and why?
- What happens as driving period increases?

Approach

Long-time Behavior of Isolated Periodically Driven Interacting Lattice Systems, *PHYSICAL REVIEW X 4, 041048 (2014)*

- Numerically constructing a finite spin-1/2 chain (with periodic boundary conditions) using QuSpin

- Calculating statistics of the averaged and Floquet Hamiltonians, describing the system.

some background on Floquet systems…

Floquet theorem

For any system, described by a time-periodic Hamiltonian, we can find a new reference frame, where the Hamiltonian, describing it is static H_F.

$$\hat{U}(t) = P(t)e^{-it\hat{H}_F}$$

$$P(t+T) = P(t)$$

$$P(lT) = \hat{1}$$

$$\hat{U}(lT) = e^{-ilT\hat{H}_F}, \text{ for } l = 1:$$

$$\hat{U}_{cycle} = e^{-iT\hat{H}_F}$$

Eigenvalue decomposition in Floquet states:

$$\hat{U}_{cycle} = \sum_n e^{-i\theta_n} |n\rangle\langle n|$$

$$\hat{H}_F = \sum_n \epsilon_n |n\rangle\langle n|$$

$$\theta_n = T\epsilon_n/\hbar$$

Floquet-Magnus Expansion, for a step drive:

$$\hat{H}_F = \sum_n^{\infty} \hat{H}_F^{(n)}$$

$$\hat{U}_{cycle} = e^{-i\frac{1}{2}T\hat{H}_1} e^{-i\frac{1}{2}T\hat{H}_0}$$

$$\hat{H}_F^{(0)} = \frac{1}{2}(\hat{H}_1 + \hat{H}_0)$$

For short driving periods T:

$$\hat{H}_F = \hat{H}_F^{(0)} + O(T)$$

the system in question…

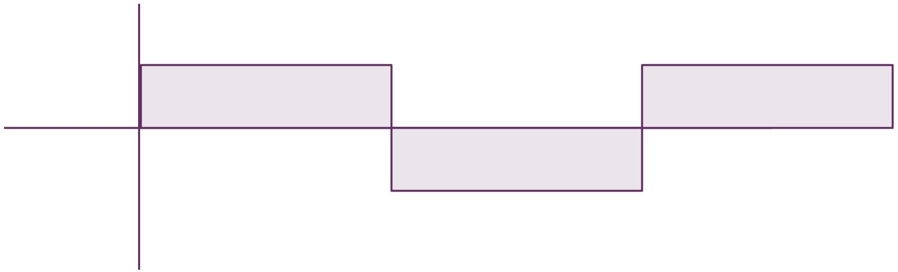
The spin chain Hamiltonian:

$$\hat{H}(t) = [J + f(t)\delta J]\hat{H}_{nn} + J' \sum_j \sigma_j^z \sigma_{j+2}^z,$$

$$\hat{H}_{nn} = \sum_j \left[\sigma_j^z \sigma_{j+1}^z - \frac{1}{2}(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y) \right]$$

Driving function:

$$f(t) \equiv \begin{cases} 1 & \text{for } N < \frac{t}{T} < N + \frac{1}{2} \\ -1 & \text{for } N + \frac{1}{2} < \frac{t}{T} < N + 1. \end{cases}$$



Numerical analysis:

- Parameters: $J = 1$, $\delta J = 0.2$, $J' = 0.8$

- System size
  $L = 18, 21, 22, 23$

- Accounting for symmetry:

  $k = 0$, $m_z = 1/3$, $p = +1$

Floquet unitary:

$$\hat{U}_{cycle} = e^{-i\frac{1}{2}T\hat{H}_-} e^{-i\frac{1}{2}T\hat{H}_+}$$

$$\Longrightarrow \quad \hat{H}_F$$

Time-averaged Hamiltonian:

$$H_F^{(0)} = \frac{1}{T}\int_0^T \mathrm{d}t\hat{H}(t)$$

$$\Longrightarrow \quad \hat{H}_{ave}$$

$$\hat{H}_F^{(0)} = \frac{1}{2}(\hat{H}_- + \hat{H}_+)$$

Figure 2: Comparing their statistics as a function of drive period T

Expectation: To coincide for small T:

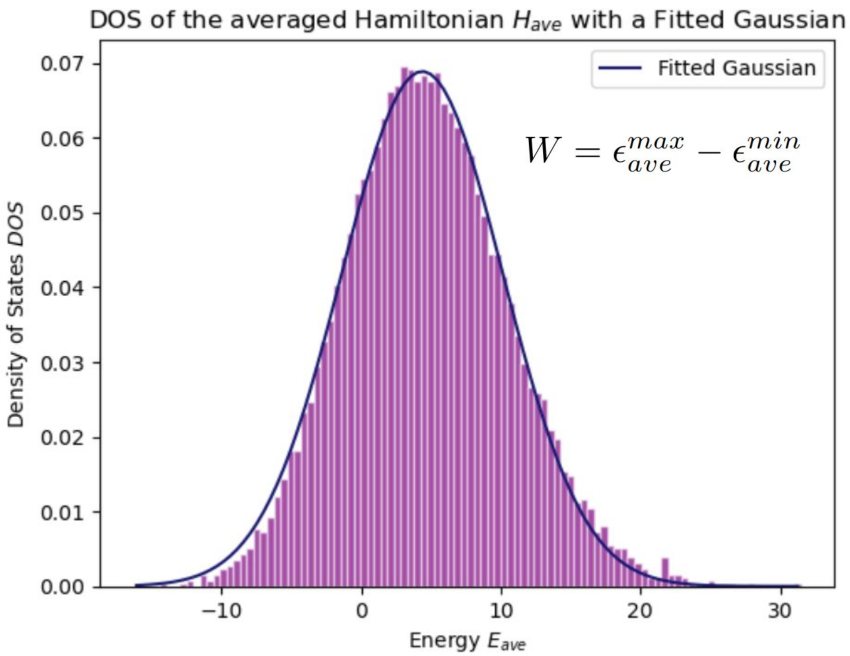
- Computed quantity: Mean level spacing ratio <r> for the phases.

$$r = \frac{\min(s_n, s_{n+1})}{\max(s_n, s_{n+1})} \in [0, 1], \qquad s_n = \epsilon_{\text{ave}}^{n+1} - \epsilon_{\text{ave}}^{n}.$$

$$r = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})} \in [0, 1], \qquad \delta_n = \theta_{n+1} - \theta_n.$$

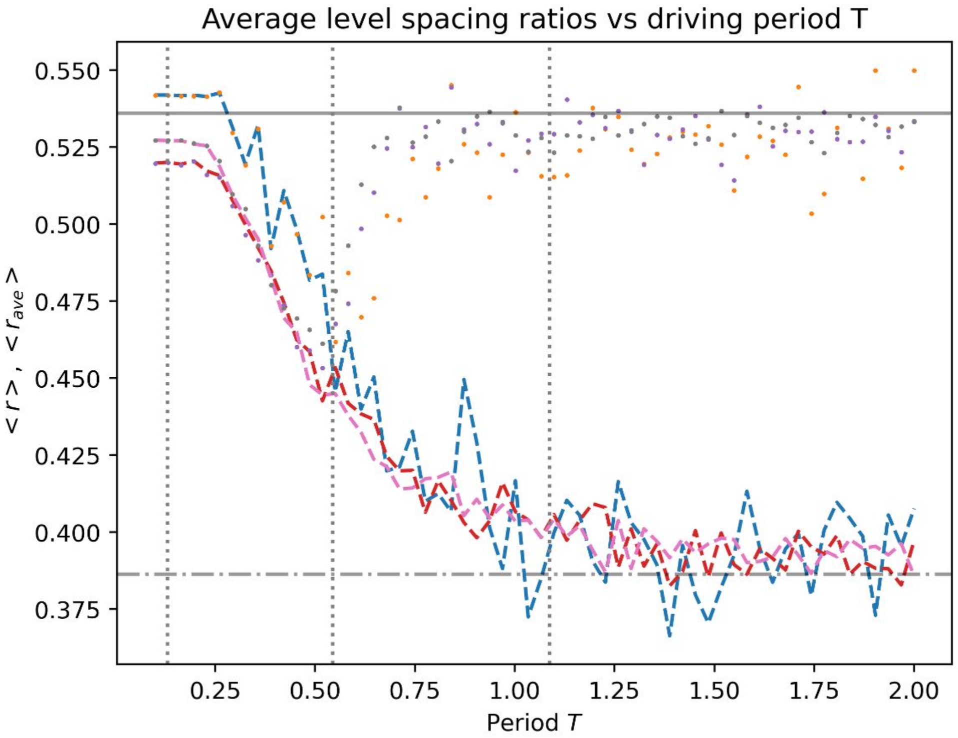
$$\theta_n = T\epsilon_n/\hbar$$

Computation:



Evaluating behaviour in three regimes and comparing to POI and GOE predictions.

$$\langle r \rangle_{\mathrm{GOE}} \approx 0.535898, \qquad \langle r \rangle_{\mathrm{POI}} \approx 0.386294.$$



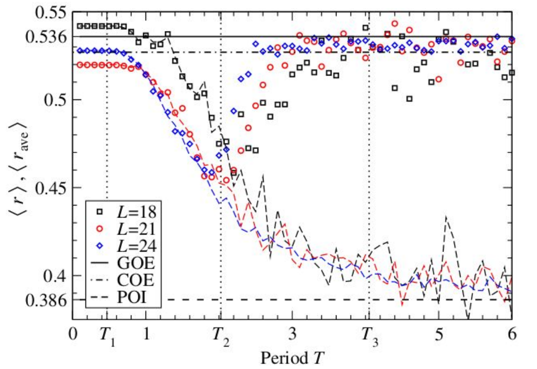
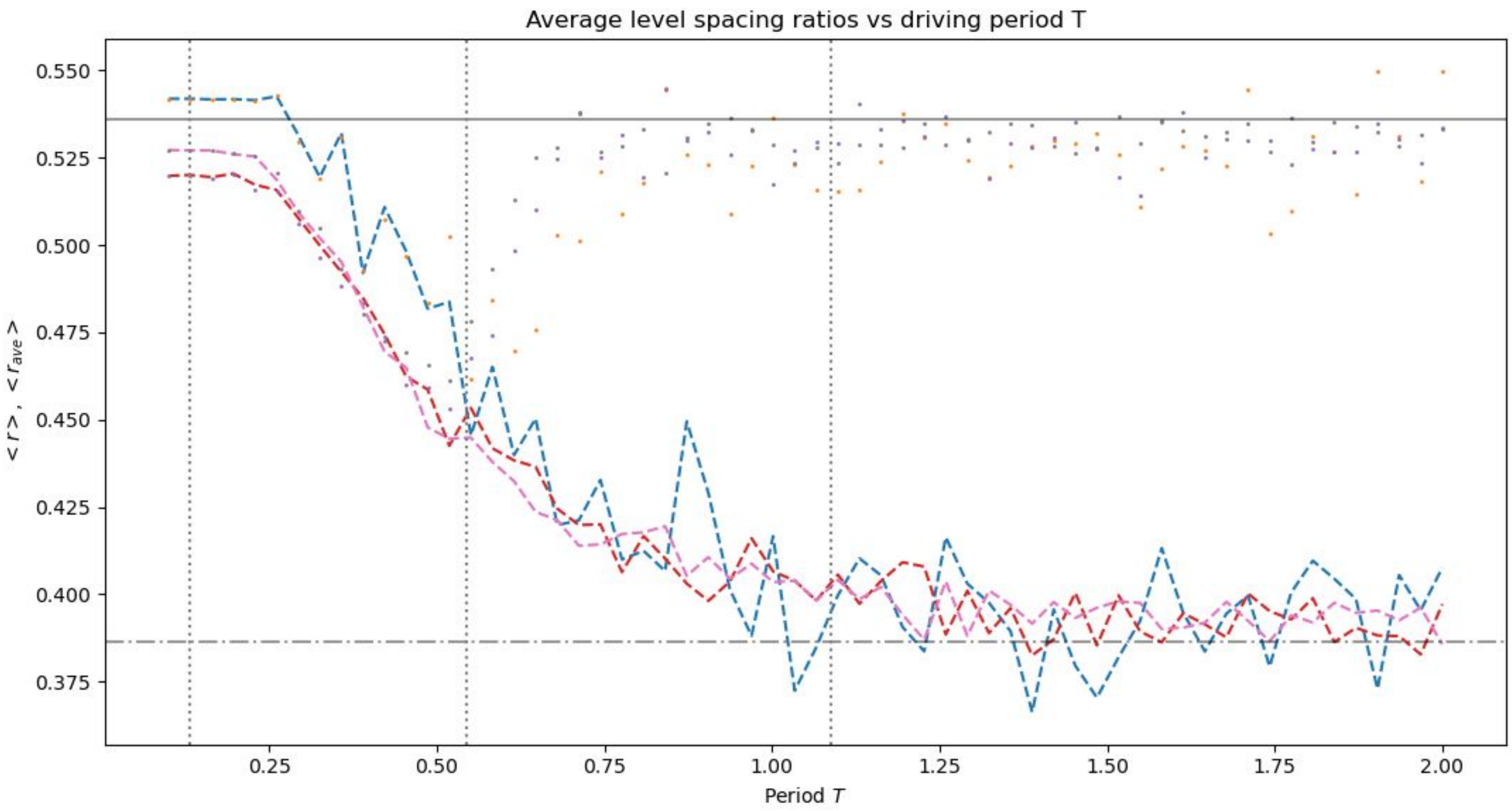


FIG. 2. Average value of r vs T. The values of $\langle r \rangle$ (symbols) and $\langle r_{\text{ave}} \rangle$ (dashed lines) are compared to the COE, GOE, and POI predictions. The COE and GOE predictions are expected to coincide in the thermodynamic limit (see Appendix B). Vertical dotted lines depict the periods $T_1 = 2\pi\hbar/W$, $T_2 = \pi\hbar/\sigma$, and $T_3 = 2\pi\hbar/\sigma$ (see text).



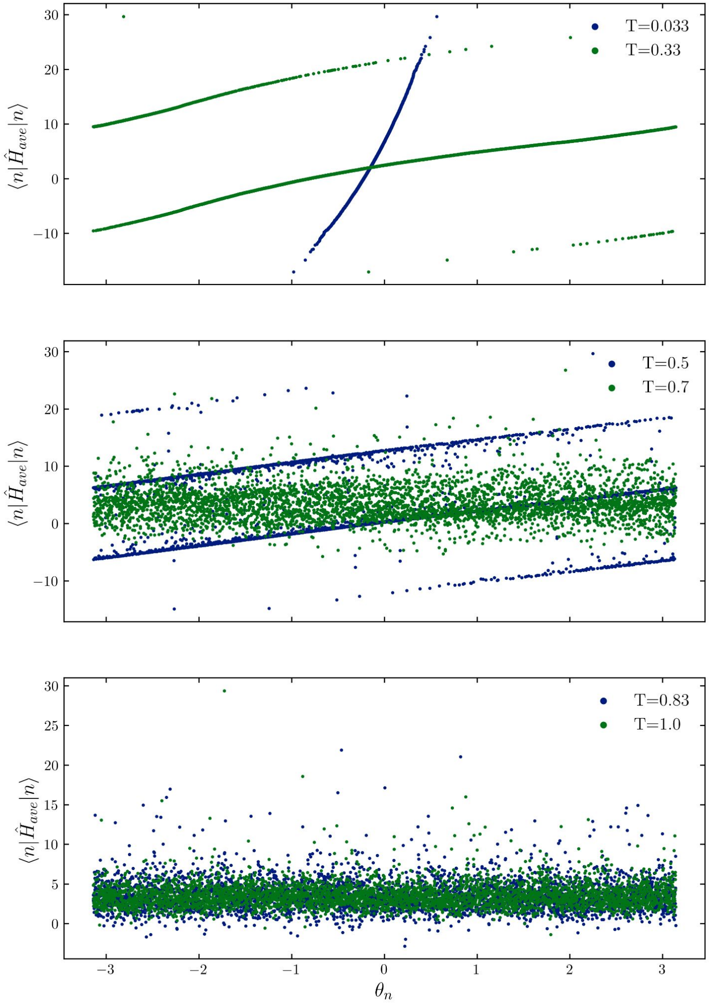
Average level spacing ratios vs driving period T
0.550
0.525
0.500
0.475
0.450
0.425
0.400
0.375
$< r >, \ < r_{ave} >$
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Period T

Figure 6: Overlap between eigenstates for different drives:

Expectation values of H_{ave} in relation to H_F eigenstates show correlation to the exact phases only for small driving periods.

Computed for L = 22.

$$\theta_n = \frac{T}{\hbar}\langle\phi_n|\hat{H}_{\text{ave}}|\phi_n\rangle$$



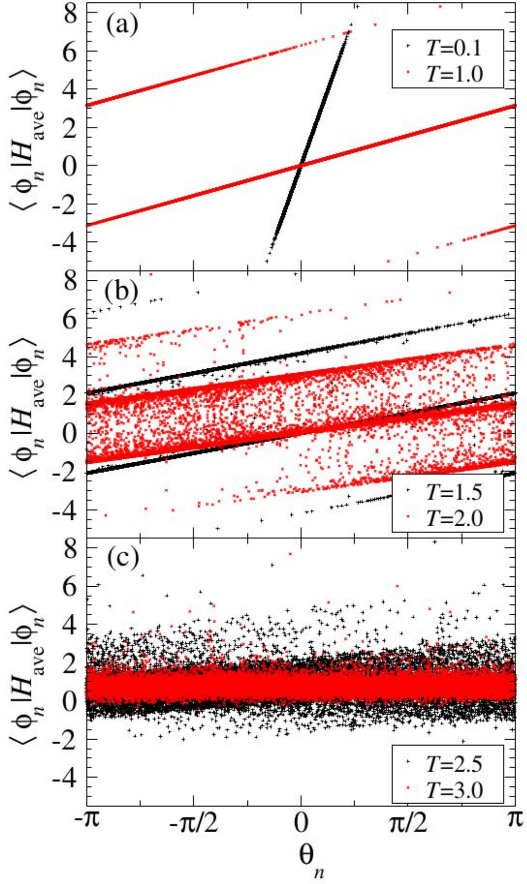


FIG. 6. Expectation values $\langle\phi_n|\hat{H}_{\mathrm{ave}}|\phi_n\rangle$ vs the exact phases θ_n folded in $[-\pi,\pi)$ for $L = 24$ and different driving periods T.

Thank you for listening.