TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

Lukas König

Random multipolar driving

Tunably slow heating

Quantum many body dynamics

Zhao, Hongzheng, et al. Physical Review Letters, vol. 126, no. 4, Jan. 2021.

What do we know already?

- **Time dependence → no time symmetry → no energy conservation → no (“infinite”) temperature**
- **With periodic time dependence:**
  - Discreet time symmetry
  - Floquet systems
  - Many-body localized states
  - Prethermalization
- **Does it have to be fully periodic?**

Interlude: where does the prethermal phase come from?

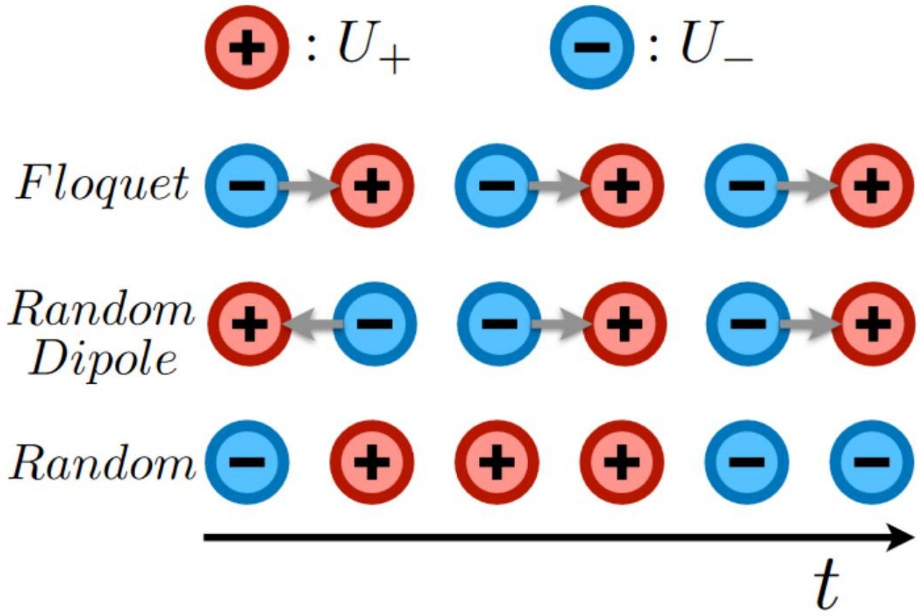
- **Floquet's theorem: there is a static (fictitious) Hamiltonian that stroboscopically describes the time evolution of a periodically driven system:** $\boldsymbol{U_1 = U_- U_+ = e^{-i2TH_{FM}}}$
- **Floquet-Magnus expansion:** $\boldsymbol{H_{FM} = \sum_{n=0}^{\infty} (2T)^n H_{FM}^{(n)}}$
- **Low order** $\boldsymbol{H_{FM}^{(n)}}$ **–> local operators**
- **FM-expansion of most physical systems diverge –> No local conserved observables left**
- **When does this divergence become relevant?**
- **Loose bound:** $\boldsymbol{\left\| U_- U_+ - e^{-iH_{FM}^{(0)} 2T} \right\| \leq (const. + \lambda T) 2T}$
- $\left\| (U_- U_+)(U_- U_+) \ldots - e^{-iH_{FM}^{(0)} t} \right\| \leq \boldsymbol{(const. + \lambda T) t}$

Simulating random drives

- **Spin chain Ising model Hamiltonian, periodic boundaries:**
- $H_{\pm} = \sum_i J_x \sigma^x{}_i \sigma^x{}_{i+1} + J_z \sigma^z{}_i \sigma^z{}_{i+1} + (B_0 \pm B_x)\sigma^x{}_i + B_z\, \sigma^z{}_i$
- **Unitaries:**
- $U_{\pm} = \exp(-iTH_{\pm})$
- **Floquet Operator** $U_1 = U_+ U_- = \exp(-i2TH_F)$
- Local $H_F^0 = (U_+ + U_-)/2$
  - Identical for U_+U_- and U_-U_+
- **Deviation from time evolution with** $\exp(-iH_F^0 t)$ **linear in t, T**
  - Long-lived prethermal plateau!
- **Look at** $\langle \boldsymbol{H_F^0} \rangle$ **and at left-chain entropy for diagnostics**

What random drives?

- **Quasiperiodic driving already shown to produce prethermalization**
- **Now: Random discreet driving (but with correlation)**
- Use time-evolution unitaries $U_\pm$ with period T
- Put them together to n-poles
  - 1-pole: U_-U_+ or U_+U_-
  - 2-pole: $U_-U_+U_+U_-$ or $U_+U_-U_-U_+$
  - …
  - ∞-pole: Thue-Morse sequence
- Apply random sequence of these unitaries

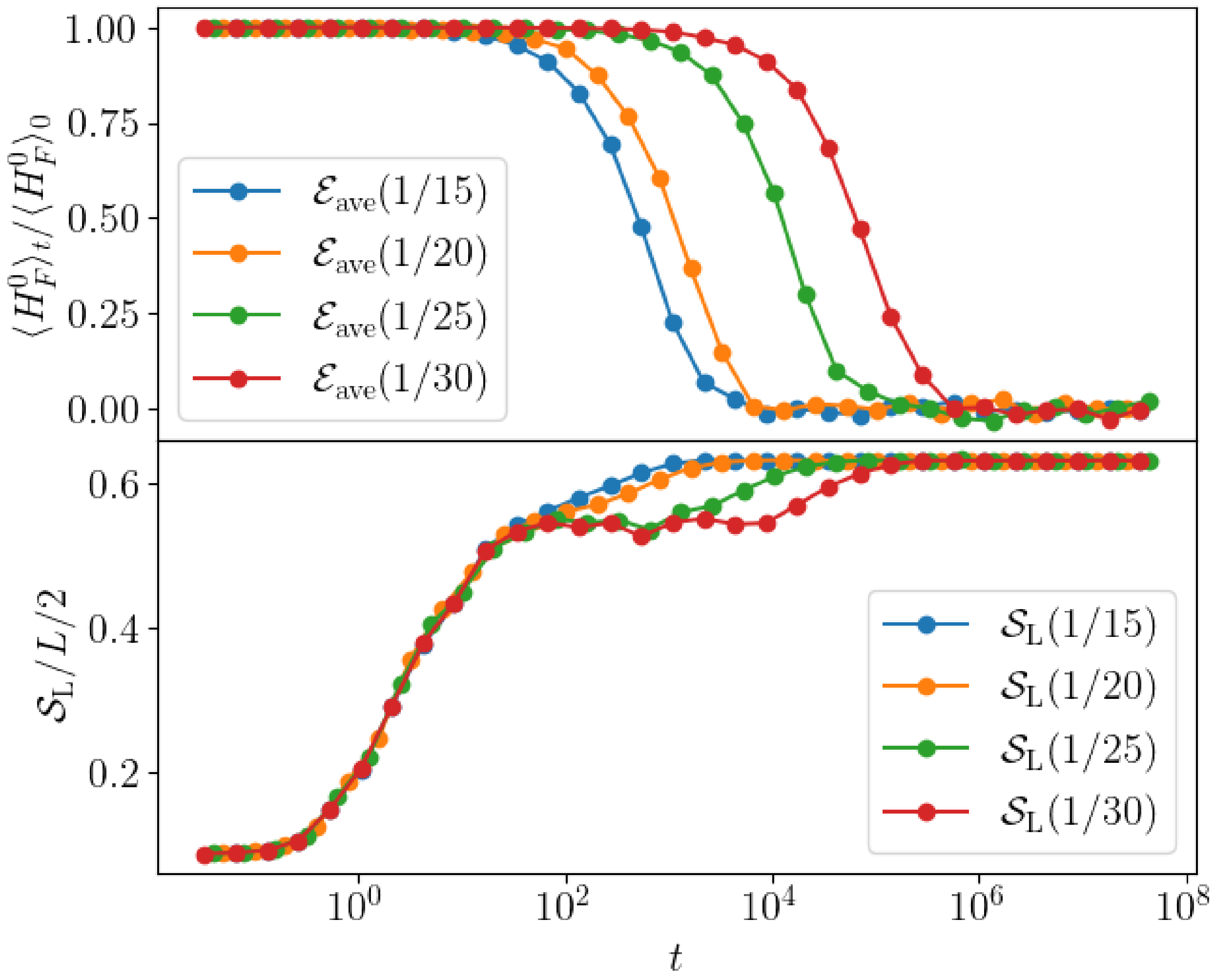


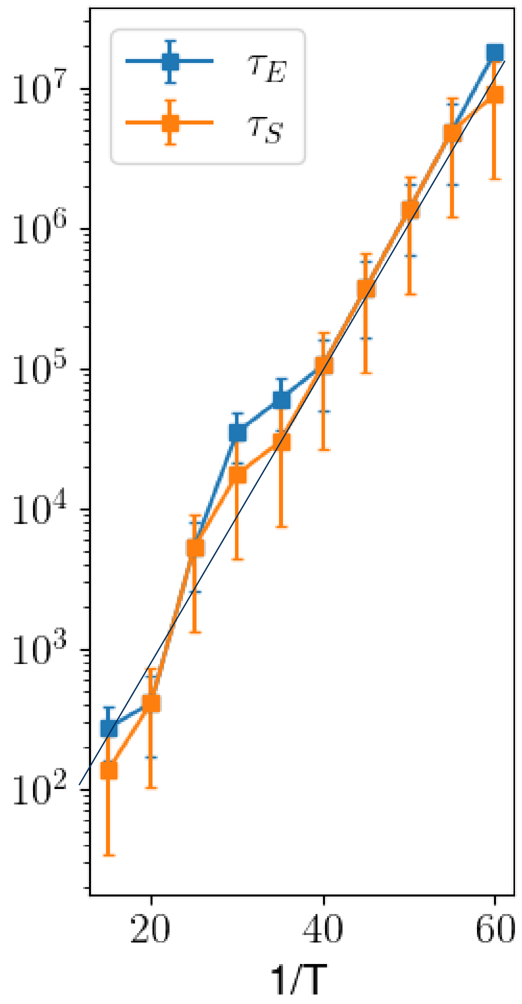
Infinite-poles

Calculation trick:

- **Introduce** $\boldsymbol{\widetilde{U_1} = U_- U_+}$
- **Then, (**with $\widetilde{U_n} = U_{n-1}\widetilde{U_{n-1}}$**) n-pole is** $\boldsymbol{U_n = \widetilde{U_{n-1}} U_{n-1}}$
- **Therefore,** $\boldsymbol{|\psi(2^n T)\rangle = U_n |\psi(0)\rangle}$
- Only linear increase in computation resources for exponential increase in time
- **However: need time evolution matrix, cannot use quspin's parallelized matrix exponential**
- RAM size limits chain size

Prethermal lifetime

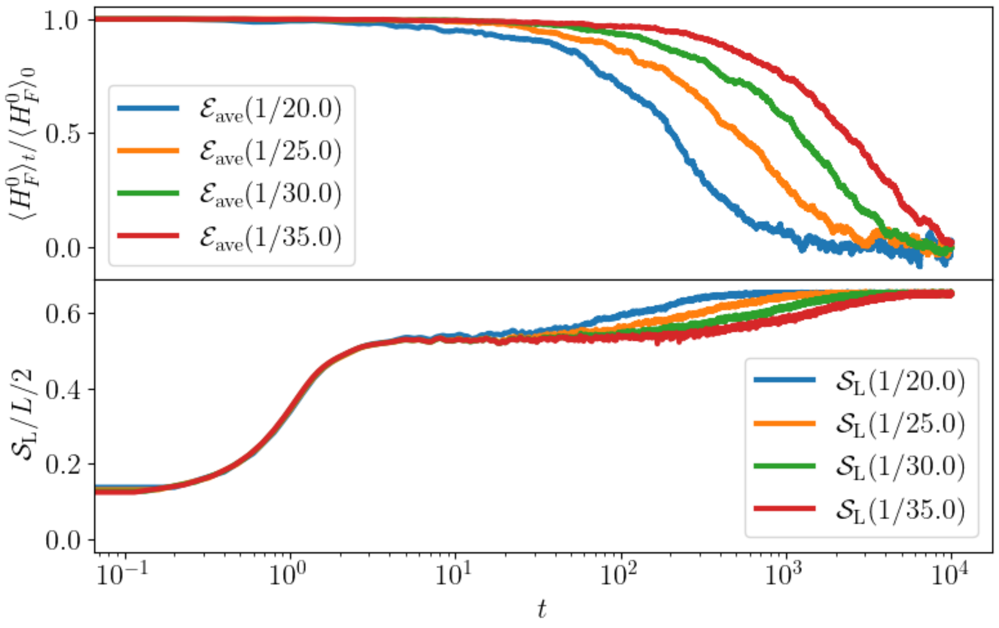


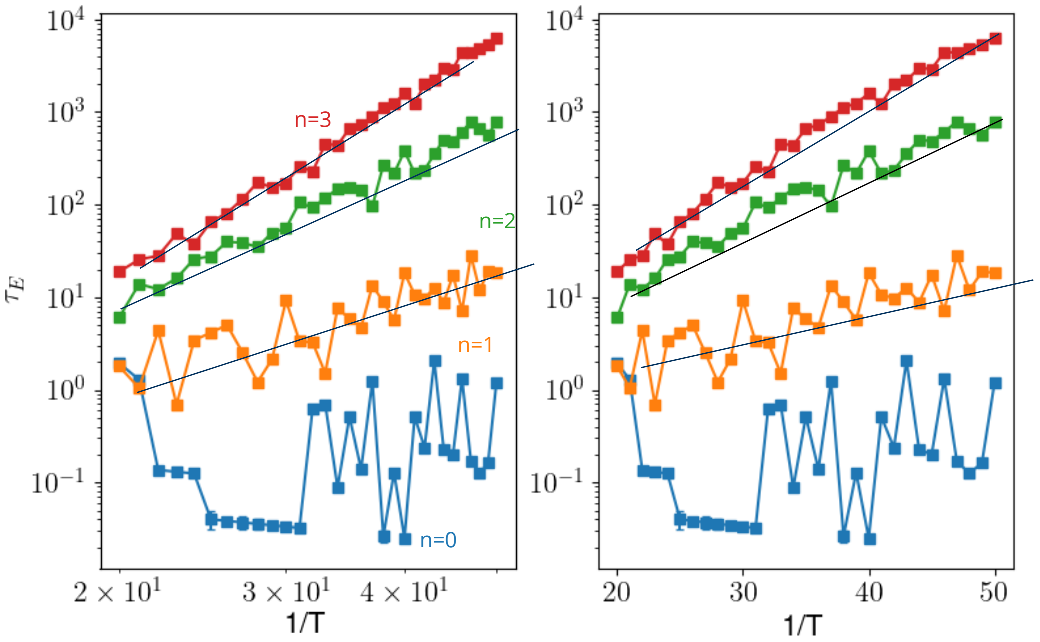


What about less structure?

Consider up to octopoles

- **Using full exponential matrix still faster than parallelized matrix exponential (for more than 0-poles)**
- **Algebraic relationship between period and lifetime $\tau_E \propto \left(\frac{1}{T}\right)^{\alpha}$**



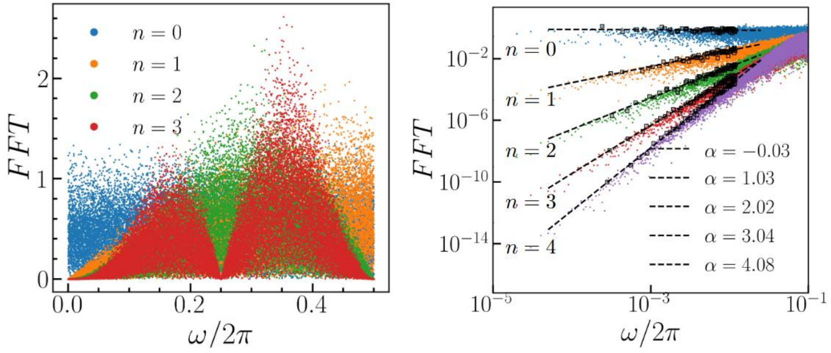


Where does the scaling come from?

- **In periodically-driven systems: driving with Fourier-series decomposition:** $g(t) = \sum_m g_m \sin\left(m \frac{2\pi}{T} t\right)$
  - Thermalization rate Γ proportional to energy absorption rate over all modes
  - Within linear response, this is constant, its inverse is the prethermalization time
  - Energy absorption rate is exponentially suppressed, proportional to ${g_m}^2$: $\Gamma = \sum_m {g_m}^2 A e^{-x\frac{2\pi}{T}/\epsilon}$

Where does the scaling come from?

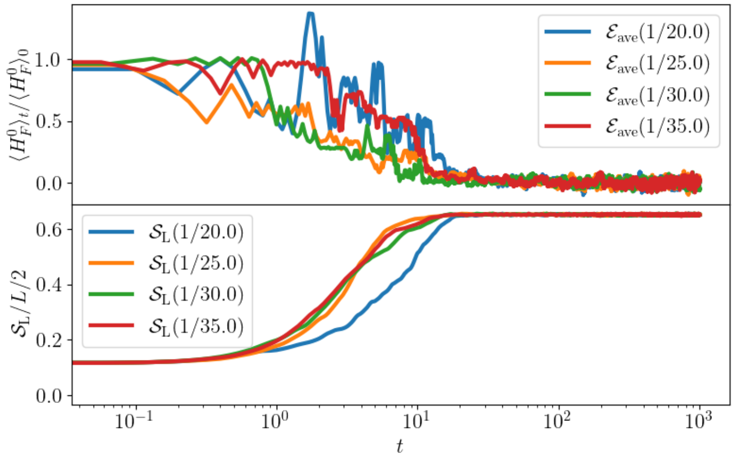
- **For our n-RMD system: continuous spectrum instead of Fourier-series:** $g(t) = \int dx\, g_x \sin\left(x \frac{2\pi}{T} t\right)$
  - $g_x \propto x^n$ (from Fourier decomposition of noise signal)
  - Algebraic or exponential lifetime behavior follows from $\Gamma \propto \int_0^\infty dx\, x^{2n} A e^{-x\frac{2\pi}{T}/\epsilon} \propto \begin{cases} (\frac{2\pi}{T})^{-(2n+1)} \\ e^{-x_0\Omega/\epsilon} \end{cases}$

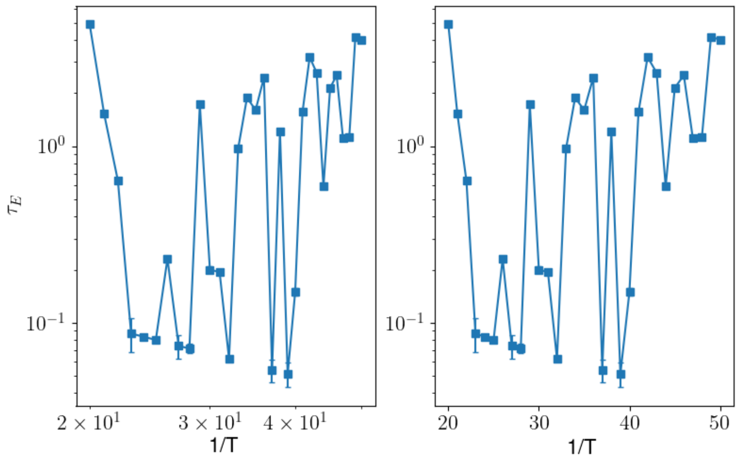


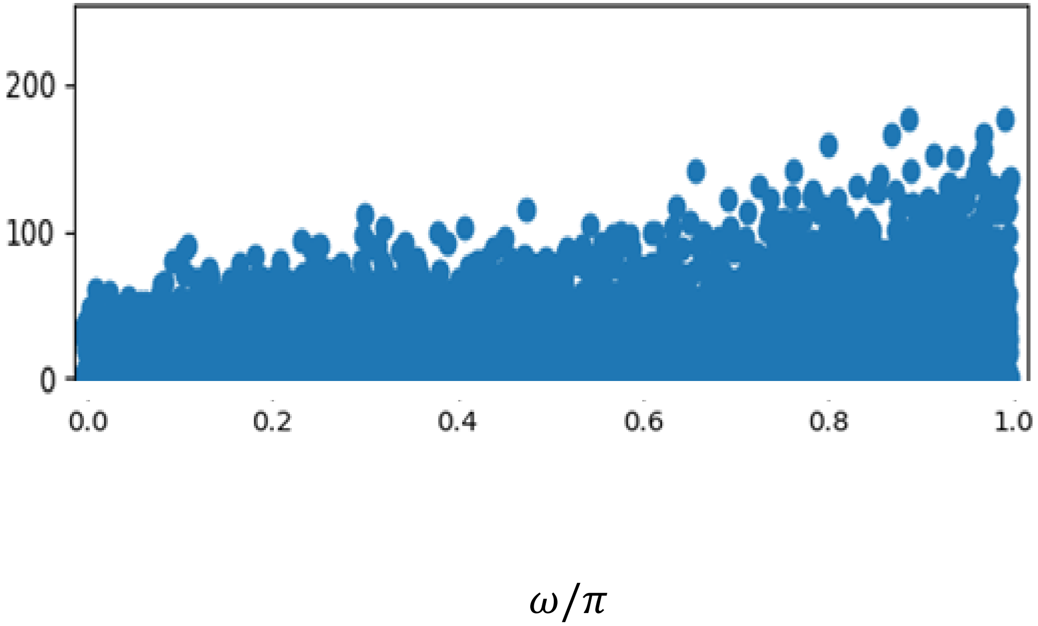
- **By agreement of numerics and analytical calculation:**

Thermalization caused by absorption of single low energy modes

Modifying the Fourier spectrum





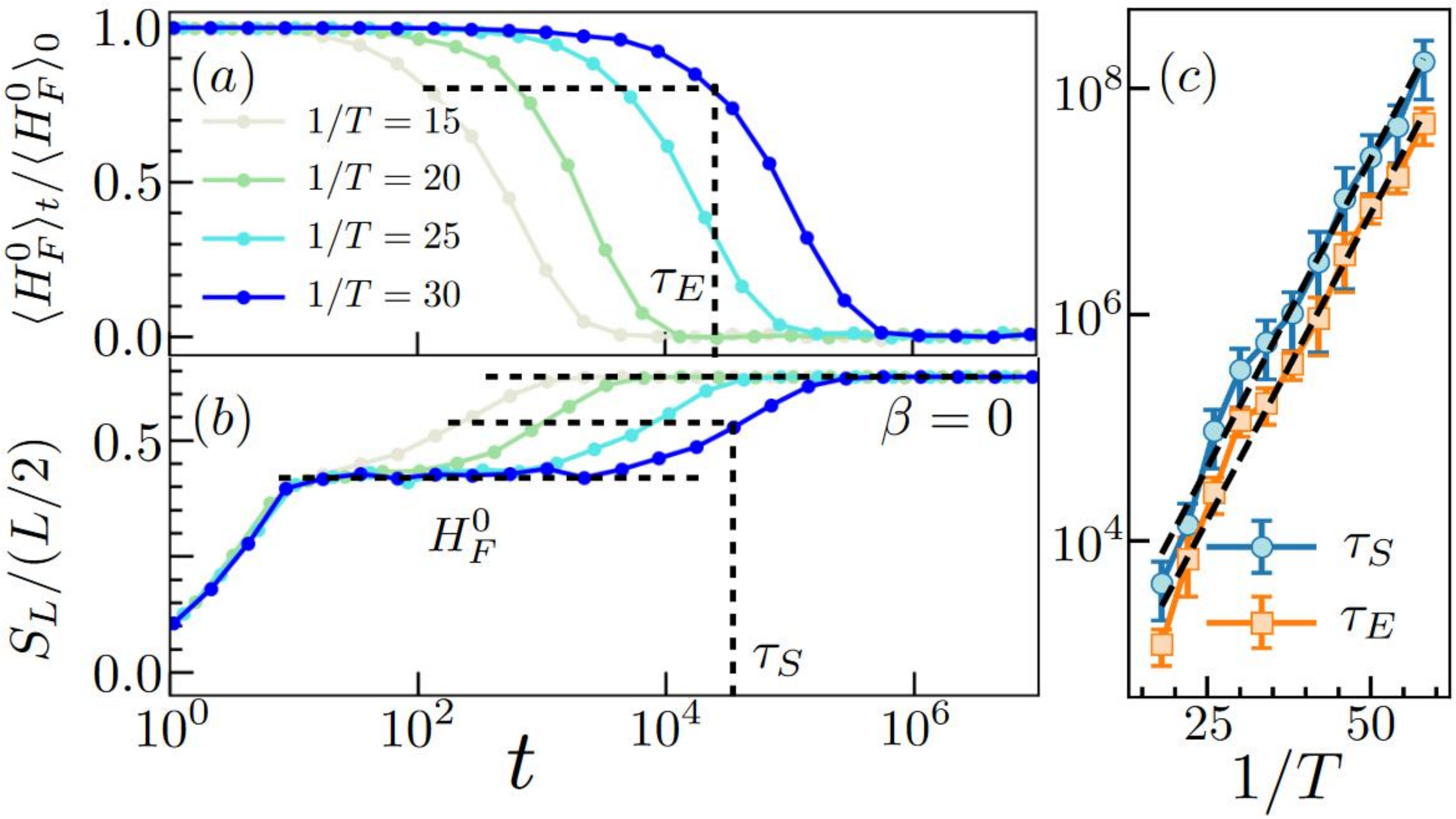


Fourier transform of noise signal

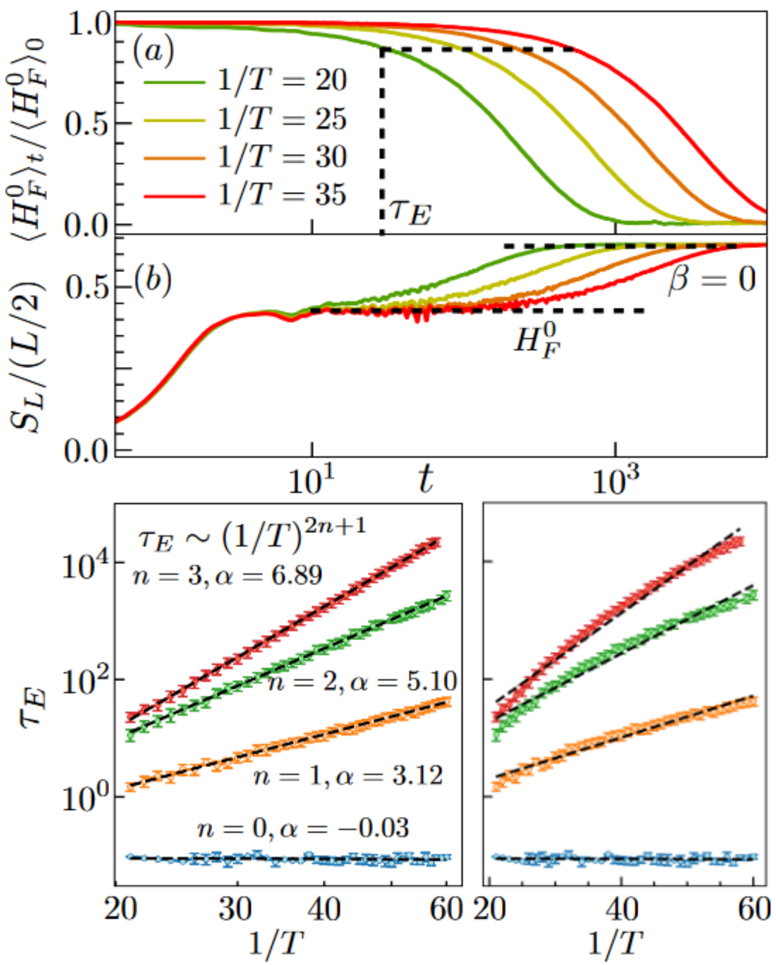
Conclusion

- **Correlated random drives also create a prethermal regime**
  - Non-equilibrium phase completely without time-transversal symmetry
- **Numerical (and analytical) confirmation of algebraic dependence on period**
- **Spectral engineering of plateau length**
  - Difficult to work from Fourier signal

Paper graphs: inifity poles



Paper graphs: n-poles



Less Dipoly stuff (U+U+U- and U-U-U+)