Driving the Transverse-Field Ising Model

The goal of this project is to study the effect of a perodic drive in the Transverse-Field Ising Model, with the aim of understanding how the exact solution and the relaxation to equilibrium is modified. Specifically, we consider the driven homogeneous Ising chain,

$$\hat{H}(t) = -\frac{1}{2}\sum_{j=1}^{L}\left[J\sigma_j^z\sigma_{j+1}^z + h(t)\sigma_j^x\right], \tag{1}$$

and study the effect of a periodic drive $h(t+T) = h(t)$. This project is based on the paper *Periodic Steady Regime and Interference in a Periodically Driven Quantum System*, Russomanno *et al.*, Phys. Rev. Lett. **109**, 257201 (2012). This project is mainly analytical, with a limited numerical component.

- Reproduce the derivation and Fig. 1 from Phys. Rev. Lett. **109**, 257201 (2012). Show how the Floquet problem again reduces to a set of two-level systems, for each of which the time-dependent Schrödinger equation can be solved numerically to obtain the many-body Floquet dynamics.

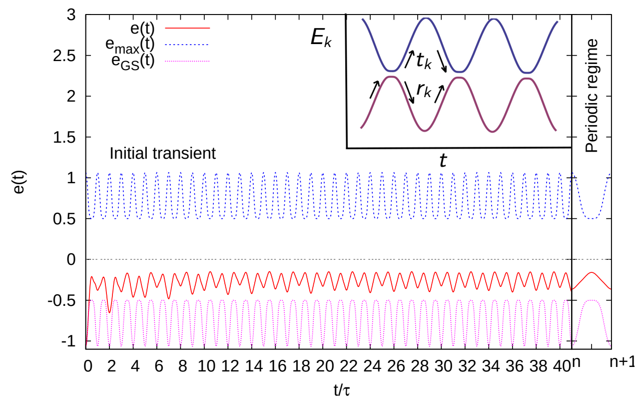

Figure 1: Dynamics of the average energy density $e(t) = \langle\hat{H}(t)\rangle$ for a periodic drive $h(t) = 1+\cos(\omega_0 t)$ with $\hbar\omega_0/J = 2$. The lower and upper curves indicate the energy of the instantaneous ground state and highest excited state. After an initial transient, the dynamics reaches a synchronized regime where the average energy density is synchronized with the drive. (For more details, see reference.)

- While the periodic drive required numerical integration of the time-dependent Schrödinger equation, we can alternatively consider a simpler two-step drive

$$h(t) = \begin{cases} h_1 & \text{for} \quad 0 \le t < T/2, \\ h_2 & \text{for} \quad T/2 \le t < T, \end{cases} \tag{2}$$

  and $h(t+T) = h(t)$. Analytically calculate the Floquet evolution operator within each 2×2 block and calculate the quasienergies as function of momentum. Find the equivalent of Fig. 1 for this drive at different driving frequencies.

- Optional goals: Compare the quasienergies/Floquet Hamiltonian with those obtained from the Magnus expansion at high driving frequencies, analytically obtain the decay rate and the synchronized state.