Eigenstate thermalization in Floquet dynamics

The goal of this project is to study the eigenstate thermalization hypothesis for the eigenstates of the Floquet Hamiltonian, and understand how Floquet resonances lead to heating. This project is based on the paper *Long-time Behavior of Isolated Periodically Driven Interacting Lattice Systems*, D'Alessio and Rigol, Phys. Rev. X **4**, 041048 (2014). This project is mainly numerical.

Specifically, we consider a driven spin chain

$$\hat{H}(t) = [1 + f(t)\delta J]\left(\sum_{j=1}^{L}\left[\sigma_j^z\sigma_{j+1}^z - \frac{1}{2}(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y)\right]\right) + J'\sum_{j=1}^{L}\sigma_j^z\sigma_{j+2}^z, \tag{1}$$

with a periodic step drive $h(t+T) = h(t)$ where $h(0 < t \leq T/2) = 1$ and $h(T/2 < t \leq T) = -1$. For this drive the Floquet Hamiltonian can be numerically constructed and its eigenstates and eigenvalue distribution can be studied. In the high-frequency limit the Floquet Hamiltonian should resemble the time-averaged Hamiltonian, which satisfies ETH. Away from the limit of an infinite-frequency drive we expect the system to eventually heat up to infinite temperature, and this project investigates how ETH needs to be modified in order to accommodate these dynamics.

- Reproduce Figs. 2 and 6 from Phys. Rev. X **4**, 041048 (2014). The first figure compares the level spacing statistics of the eigenspectrum of the Floquet Hamiltonian with the statistics of the corresponding expectation value of the time-averaged Hamiltonian. These coincide as $T \to 0$, but for increasing driving period these no longer agree, with the eigenphases staying close to the random matrix prediction, whereas the expectation values start to behave as (Poissonian) random variables. The second figure indicates how this process can be observed and understood through the 'folding' of the eigenspectrum leading to the appearance of Floquet resonances.

- Optional goals: Check the failure of Floquet ETH in the driven Transverse-Field Ising Model. Characterize the distribution of the off-diagonal matrix elements and verify that these behave as Gaussian random variables. Numerically study the (pre)thermalization dynamics as a function of driving frequency. How does the time scale for prethermalization depend on the driving frequency?

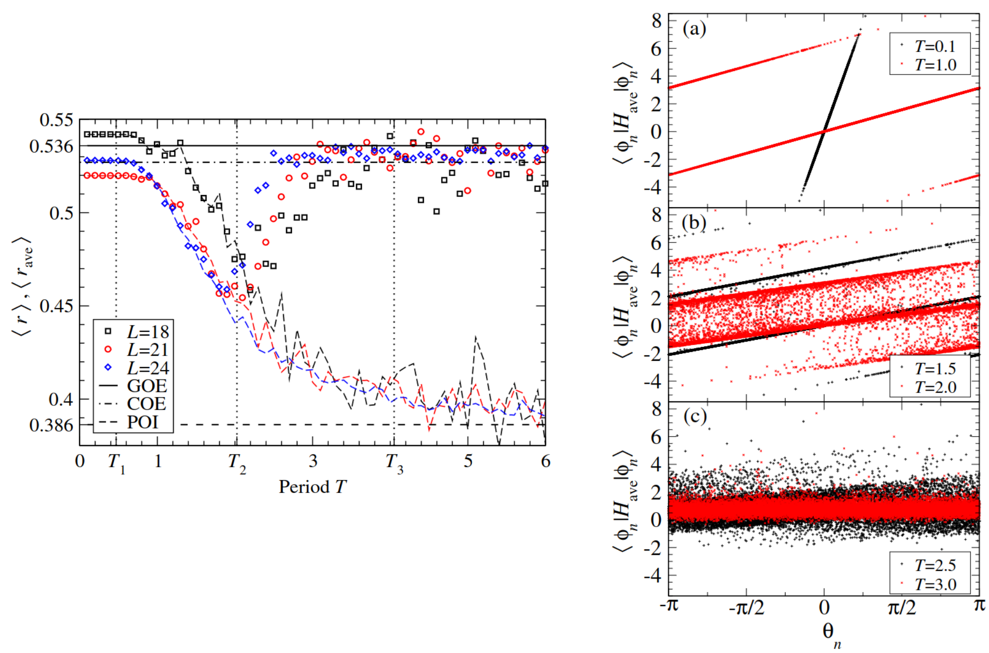

Figure 1: (Left) Average level spacing ratio $\langle r \rangle$ for the eigenspectrum of the Floquet Hamiltonian and the corresponding expectation values of the time-averaged Hamiltonian as a function of driving frequency, compared to the COE, GOE, and POI predictions. (Right) Expectation values of the time-average Hamiltonian w.r.t. the eigenstates $|\phi_n\rangle$ of the Floquet Hamiltonian as a function of the corresponding eigenphases θ_n. See reference for details.